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Statistical Methods for Estimating the Uncertainty in the Best Basis Inventories

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Abstract: This document describes the statistical methods used to determine sample-based uncertainty estimates for the Best Basis Inventory (BBI). For each waste phase, the equation for the inventory of an analyte in a tank is Inventory (Kg or Ci) = Concentration x Density x Waste Volume. The total inventory is the sum of the inventories in the different waste phases. Using tanks sample data; statistical methods are used to obtain estimates of the mean concentration of an analyte the density of the waste, and their standard deviations. The volumes of waste in the different phases, and their standard deviations, are estimated based on other types of data. The three estimates are multiplied to obtain the inventory estimate. The standard deviations are combined to obtain a standard deviation of the inventory. The uncertainty estimate for the Best Basis Inventory (BBI) is the approximate 95% confidence interval on the inventory.

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STATISTICAL METHODS FOR ESTIMATING THE UNCERTAINTY IN THE BEST BASIS INVENTORIES

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SUMMARY

This document describes the statistical methods used to determine sample-based uncertainty estimates for the Best Basis Inventory (BBI). For each waste phase, the equation for the inventory of an analyte is

$$\text{Inventory (kg or Ci)} = \text{Concentration} \times \text{Density} \times \text{Waste Volume}$$

The total inventory is the sum of the inventories in the different waste phases.

Using tank sample data, statistical methods are used to obtain estimates of the mean concentration of an analyte, the density of the waste, and their standard deviations. The volumes of waste in the different phases, and their standard deviations, are estimated based on other types of data. The three estimates are multiplied to obtain the inventory estimate. The standard deviations are combined to obtain a standard deviation of the inventory. The uncertainty estimate for the BBI is the approximate 95% confidence interval on the inventory.

An example of the BBI uncertainties is reported in Section 3.0. The uncertainty results for 33 tanks are available on the Tank Waste Information Network System (TWINS3) database at

http://twins.pnl.gov:8001/datamenu.htm

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TERMS

BBI	Best Basis Inventory
C	analyte concentration
$\overline{C}$	estimate of analyte concentration
Ci	Curie
CI	confidence interval
D	waste density
$\overline{D}$	estimate of waste density
DST	Double-Shell Tank
FY	fiscal year
g/mL	grams per milliliter
I	inventory
$\hat{I}$	estimate of inventory
kg	kilogram
kL	kiloliter
LL	lower limit to an approximate 95% confidence interval
REML	restricted maximum likelihood method
RSD	relative standard deviation
SST	Single-Shell Tank
SD	standard deviation
TCD	Tank Characterization Database
TCR	Tank Characterization Report
TIR	Tank Interpretive Report
TWINS	Tank Waste Information Network System
UL	upper limit to an approximate 95% confidence interval
V	waste volume
$\hat{V}$	estimate of waste volume
%	percent
μg/g	microgram per gram
μg/mL	microgram per milliliter

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1.0 INTRODUCTION

This document describes the statistical methods used to determine sample-based uncertainty estimates for the Best Basis Inventory (BBI). The statistical methods were applied to sample-based data for tanks with a tank characterization report (TCR) written in fiscal year (FY) 2000, tanks with waste transfers in FY 2000, and Phase I retrieval tanks. Table 1-1 lists the specific tanks according to the three classes. Uncertainty estimates were not computed for six of the transfer tanks in this list because the inventories were not sample-based.

Table 1-1. Candidate Tanks for Best-Basis Inventory Uncertainty Estimates.

TCR	Transfer Tanks		Phase I Tanks
241-AN-101[1]	241-A-101	241-S-106	241-AN-102[1]
241-AN-102[1]	241-AN-101[1]	241-S-108[2]	241-AN-103
241-AN-107[1]	241-AP-103	241-SX-104[1]	241-AN-104
241-AW-101[1]	241-AP-104[2]	241-SX-106	241-AN-105
241-AZ-101[1]	241-AP-105[2]	241-SY-101[1]	241-AN-107[1]
241-AZ-102[1]	241-AP-106[2]	241-SY-102	241-AW-101[1]
241-C-104[1]	241-AP-107[2]	241-T-104	241-AZ-101[1]
241-S-103[1]	241-AP-108	241-T-110	241-AZ-102[1]
241-SX-104[1]	241-AW-102	241-U-102[1]	241-SY-101[1]
241-SY-101[1]	241-AW-106[2]	241-U-103[1]	241-SY-102[1]
241-TX-113	241-AY-101[1]	241-U-105[1]	241-AP-101
241-TX-118	241-AY-102	241-U-109[1]	241-AY-101[1]
241-U-102[1]	241-C-106		241-AY-102[1]
241-U-103[1]	241-S-102		241-C-104[1]
241-U-105[1]	241-S-103[1]		241-C-106[1]

Notes:

[1] tank on multiple lists
[2] inventory not sample-based

The statistical techniques and the assumptions used to determine the uncertainty estimates for the BBI are described in Section 2.0. Section 3.0 contains an example of the computations needed to determine the uncertainty in the BBI for a specific tank and analyte.

Uncertainty estimates were previously determined for all of the Best-Basis tank inventories (Ferryman et al. 1998). Since the original uncertainty estimates, a number of transfers have occurred, additional samples have been obtained, and Best-Basis Inventories have been revised. This document, presents uncertainty estimates for selected sample-based BBI values as of June 30, 2000, in partial fulfillment of Performance Incentive 1.1.1 part 7, and the U.S. Department of Energy, Office of River Protection (DOE-ORP) milestone 2.3.3. The DOE-ORP milestone reads:

> "Existing uncertainty estimates shall be updated for the 13 tanks with complete TIRs and for Phase I retrieval tanks to reflect new sampling information. Uncertainty estimates for waste transfer tanks shall be completed in accordance with the June 30, 2000 cut-off date. Uncertainty estimates shall be posted on TWINS3. BBI estimates shall be calculated on a waste phase basis where supportive sampling information exists."

Best-Basis inventory estimates are sample-based, engineering-based, or model-based. Although uncertainty estimates were previously determined for sample-, engineering-, and model-based data, it was determined that sufficient information would be provided and resources better utilized by updating only sample-based uncertainty estimates for the current effort. The uncertainty results computed using the statistical methods described in this document are available on the TWINS3 database at

http://twins.pnl.gov:8001/datamenu.htm

In the future, contingent on priority and resources, inventory uncertainties for sample-based data will be updated for all tanks.

2.0 STATISTICAL TECHNIQUES FOR BEST-BASIS INVENTORY UNCERTAINTY

2.1 INVENTORY ESTIMATE FOR A WASTE PHASE

For each waste phase, the equation for the inventory of an analyte in a tank is

$$\text{Inventory (kg or Ci)} = \text{Concentration} \times \text{Density} \times \text{Waste Volume, or}$$

$$I = C \times D \times V.$$

(The term density will be used to denote "Bulk Density" or "Specific Gravity" depending on the waste phase solid or liquid.) The statistical methods for estimating the inventory "I" and the methods for constructing an approximate confidence interval for "I" are described in the following paragraphs. The approximate 95% confidence interval on the inventory is the uncertainty for the BBI.

2.2 SUMMARY OF THE STATISTICAL TECHNIQUES

The steps followed to obtain an estimate of the inventory of an analyte and the standard deviation of the estimates (the uncertainty) are as follows.

- The total volume of waste in a tank is separated into distinct phases; e.g., saltcake, supernatant, etc.

For each waste phase and analyte:

- Obtain an estimate of the analyte concentration, and waste density, and the standard deviation of the estimates, using tank sample data.
- Using other types of information, obtain an estimate of the volume of the waste phase and the "standard deviation" of the estimate.

The inventory of an analyte in a waste phase is the product of the three estimates. The standard deviation of the inventory is a complicated combination of the three individual standard deviations.

The total inventory of an analyte in a tank is obtained by adding the inventories in the different waste phases. The standard deviation of the total inventory is also a complicated combination of the individual standard deviations.

The estimates of analyte inventory, and the standard deviation of the estimate, are not based on specific probability distributions such as the normal distribution. However, the 95% confidence interval on the inventory is a confidence interval that is analogous to the confidence interval for a mean based on the normal distribution with a known variance.

Note that the CI on the inventory uses both quantitative and qualitative information. Consequently, the CI it does not have the same interpretation as a CI based on core sample data.

The computer program S-PLUS 2000™ (S-PLUS 2000) was used to estimate the required means and standard deviations. It was also used to combine the concentration estimates, the density estimates, and volume estimates into the final inventory uncertainty estimates.

2.3 SPECIFIC STATISTICAL TECHNIQUES

Based on the core sample data, available from the Tank Characterization Database (TCD) Tank Waste Information Network System (TWINS), Version 3.28, the restricted maximum likelihood method (REML) was used to estimate the mean concentration and standard deviation of the mean ($\overline{C}$ and $SD(\overline{C})$) for the analytes in each waste phase. These estimates are reported in the Tank Characterization Reports (TCRs) and stored in the "Means and Variances" template (Means and Variances Standard Report (TWINS configuration identifier - SR08)).

Some of the observations for a given analyte are below the detection limit. The following method was used to incorporate such observations. If more than 50% were above the detection limit, then the detection limit was used as a quantitative value in computing the mean and variance. If more than 50% were below the detection limit, then no uncertainty estimates were computed. In this case a default relative standard deviation (RSD) of 100% was used.[1] Also, if there was only one observation a default RSD of 100% was used.

Using data from TCD, an estimate of the mean density $\overline{D}$ and standard deviation of the mean ($SD(\overline{D})$) were obtained for each type of tank, single-shell tank (SST) or double-shell tank (DST), and waste phase, liquid or solid. All of the bulk density and specific gravity observations in TCD were obtained. The observations were separated into four classes by type of tank (SST or DST) and waste phase (liquid or solid).

For each of the four classes, a one-way analysis of variance model was fit to the data. The classification variable was the tank name. The results of the analysis of variance are given in Table 2-1. Specifically, the analysis of variance was used to estimate the "pooled" within tank variance. This variance is often called the "error variance." This variance is the S^2(error) in Table 2-1. In this table, the column Num.Tanks reports the number of tanks with Bulk Density or Specific Gravity observations that were used in the computations, and the Mean of Means column is the un-weighted average of the Bulk

[1] A justification for using 100% as the default value of the RSD is as follows. Let the random variable X be defined on the interval [0,1] and let its mass be concentrated at 0 and 1 with probabilities p and 1-p; i.e., "0" represents below the detection limit and "1" above the detection limit. The mean of X is p and the variance is p(1-p). The maximum value of the variance occurs when p=0.5. Consequently the variance is less than $(0.5)^2$ and the RSD is less than 100%.

Density or Specific Gravity tank means. The RSD is the square root of S^2(error) divided by the Mean of Means, times 100. These RSDs were used for each of the four tank/waste phase classifications.

Table 2-1. Relative Standard Deviations (RSDs) for Bulk Density and Specific Gravity by Type of Tank and Waste Phase.

Tank and Phase	Method	Num.Tanks	Mean of Means	S^2(error)	RSD(%)
SST, Liquid	Specific Gravity	52	1.3293	0.006146	5.90%
SST, Solid	Bulk Density	60	1.5682	0.014001	7.55%
DST, Liquid	Specific Gravity	26	1.2036	0.009642	8.16%
DST, Solid	Bulk Density	18	1.5047	0.009553	6.50%

An estimate of the volume of waste $(\hat{V})$ was obtained for each waste phase. The methods for estimating the waste volumes and the "standard deviation" or the uncertainty in the volume estimates are given in Yorgesen (2000). This reference contains the estimates of the volumes of waste $(\hat{V})$ and the "standard deviations" for the waste volumes $(SD(\hat{V}))$.

The estimate of the inventory in a waste phase is

$$\hat{I} = \bar{C} \times \bar{D} \times \hat{V} \tag{2-1}$$

The relative standard deviation (RSD) of the inventory estimate is

$$RSD(\hat{I}) = \frac{SD(\hat{I})}{\hat{I}}. \tag{2-2}$$

(An RSD is the standard deviation divided by the mean.).

If it is assumed that the three variables $\bar{C}$, $\bar{D}$, and $\hat{V}$ are independent of each other, then the RSD^2 of the product (Hogg and Craig 1971, page 168) is

$$\begin{aligned} RSD^2(\hat{I}) = {} & RSD^2(\bar{C}) + RSD^2(\bar{D}) + RSD^2(\hat{V}) \\ & + RSD^2(\bar{C}) \times RSD^2(\bar{D}) + RSD^2(\bar{C}) \times RSD^2(\hat{V}) + RSD^2(\bar{D}) \times RSD^2(\hat{V}) \\ & + RSD^2(\bar{C}) \times RSD^2(\bar{D}) \times RSD^2(\hat{V}). \end{aligned} \tag{2-3}$$

In some cases, the four product terms in this equation are neglected and the RSD^2 is, approximately,

$$RSD^2(\hat{I}) \cong RSD^2(\bar{C}) + RSD^2(\bar{D}) + RSD^2(\hat{V}) \tag{2-4}$$

This approximation is the commonly quoted equation for the RSD^2 of a product of three independent random variables. The standard deviation of the inventory estimate is

$$SD(\hat{I}) = \hat{I} \times RSD(\hat{I}), \text{ where } RSD(\hat{I}) = \sqrt{RSD^2(\hat{I})}. \qquad (2\text{-}5)$$

The approximation to the RSD, given in Equation 2-4, was used to compute the standard deviation of the inventory estimate.

For each analyte, the estimates of the mean concentration and standard deviation of the mean ($\overline{C}$ and $SD(\overline{C})$) are based on samples from the tank. Likewise, the mean density and its standard deviation ($\overline{D}$ and $SD(\overline{D})$) are based on samples from a tank. These estimates are reported in the TCRs. These estimated are based on quantitative observations and standard statistical methods.

An estimate of the volume of waste ($\hat{V}$) was obtained for the various waste phases and for the total. The estimates of the uncertainty in the volumes are qualitative. Consequently, the estimate of the volume of waste ($\hat{V}$) and an uncertainty or "standard deviation" for the waste volume ($SD(\hat{V})$) do not have the same statistical properties as the analyte concentrations and waste density estimates. For example, there are no "degrees of freedom" associated with the waste volume estimate. The waste volume estimates use in these computations, are reported in Yorgesen (2000).

Since the estimate of the inventory is a combination of quantitative and qualitative information, a confidence interval for the mean, based on Student's t distribution or any other probability distribution, is not appropriate. An approximate 95% confidence interval (CI) for the inventory, I, is the estimate of the inventory plus or minus 2 times the standard deviation of the estimate.

That is, an approximate 95% CI is the interval (LL, UL) where the lower (LL) and upper (UL) limits are

$$LL = \hat{I} - 2 \times \hat{I} \times RSD(\hat{I}) \text{ and } UL = \hat{I} + 2 \times \hat{I} \times RSD(\hat{I}). \qquad (2\text{-}6)$$

or, in terms of standard deviations,

$$LL = \hat{I} - 2 \times SD(\hat{I}) \text{ and } UL = \hat{I} + 2 \times SD(\hat{I}). \qquad (2\text{-}7)$$

This interval is the uncertainty estimate for a waste phase for the BBI. The factor "2 times the standard deviation of the estimate" in the CI is analogous to the factor "1.96 times the standard deviation of the mean" for a CI on the mean based on a normal distribution with a known variance. Since the CI uses both quantitative and qualitative information it does not have the same interpretation as a CI based on core sample data.

2.4 COMBINED INVENTORY ESTIMATES

Let $\hat{I}_1, \hat{I}_2, \ldots, \hat{I}_k$ and $SD(\hat{I}_1), SD(\hat{I}_2), \ldots, SD(\hat{I}_k)$ be the inventory estimates and standard deviations of the estimates for each of the waste phases. The combined inventory estimate is

$$\hat{I} = \sum_{i=1}^{k} w_i \times \hat{I}_i \qquad (2\text{-}8)$$

where $w_1, w_2, \ldots, w_k$ are known "weights" (unless noted otherwise, the weights are all equal to one). The standard deviation of the combined estimate is the square root of the variance of the estimate. In this case,

$$\text{var}(\hat{I}) = \sum_{i=1}^{k} w_i^2 \times SD^2(\hat{I}_i) + 2 \times \sum_{i<j} w_i \times w_j \times \text{Cov}(\hat{I}_i, \hat{I}_j) \qquad (2\text{-}9)$$

where $\text{Cov}(\hat{I}_i, \hat{I}_j)$ is the covariance between the inventories in the i^{th} and j^{th} waste phases. In general, the covariance is unknown. However, since the correlation between two variables is less than one in absolute value (Hogg and Craig 1971, page 75), the covariance is less than the product of the two standard deviations. That is

$$\text{Cov}(\hat{I}_i, \hat{I}_j) \leq SD(\hat{I}_i) \times SD(\hat{I}_j) \qquad (2\text{-}10)$$

consequently, the variance of the inventory has an upper bound of

$$\begin{aligned} \text{var}(\hat{I}) &\cong \sum_{i=1}^{k} w_i^2 \times SD^2(\hat{I}_i) + 2 \times \sum_{i<j} w_i \times w_j \times SD(\hat{I}_i) \times SD(\hat{I}_j) \\ &= \left\{ \sum_{i=1}^{k} w_i \times SD(\hat{I}_i) \right\}^2 \end{aligned} \qquad (2\text{-}11)$$

In this case, the standard deviation of the combined inventory estimate becomes

$$SD(\hat{I}) \cong \sum_{i=1}^{k} w_i \times SD(\hat{I}_i). \qquad (2\text{-}12)$$

The approximate 95% CI on the combined inventory is the interval (LL, UL) where

$$LL \cong \hat{I} - 2 \times SD(\hat{I}) \text{ and } UL \cong \hat{I} + 2 \times SD(\hat{I}). \qquad (2\text{-}13)$$

Appendix A contains the Review Checklist from a review of the statistical methods used to estimate the uncertainty for the BBI.

3.0 AN EXAMPLE

This section contains an example showing the steps followed to provide an uncertainty estimate for a BBI. The example is for aluminum, Al, in tank 241-U-109.

Tables 3-1 and 3-2 were obtained from TCD by requesting a summary of the sample-based computations used to estimate the kg of Al in 241-U-109. Table 3-1 lists the two waste phases (solid and liquid) in column one. The second column contains a more detailed description of the waste phases. The third column is the BBI in kg Al for the waste phases. The fourth column lists the concentration of Al in the phases; note that the units are μg/g even for results based on liquid samples. The equations describing how the data were combined to estimate an inventory are in column five.

Table 3-2 is a continuation of Table 3-1 with the waste phases given in the first column. The second column lists the volumes, kL, of the waste phases, and the third column gives the sample based density estimates. Columns five and six are the sample-based estimates of the mean Al concentrations and standard deviations of the mean (SD(Mean)). These are the REML estimates stored in the Means and Variances standard report. Note that in these two columns the units for the liquid sample results are μg/mL and that they have been converted to μg/g in column four of Table 3-1.

Table 3-3 repeats the columns with the concentration estimates, the volumes, and the densities for the different phases that were given in Tables 3-1 and 3-2. However, note that the rows have been rearranged with results based on solid sample first followed by those for liquid samples. To the right of each of the columns with the concentration estimates, the volumes and the densities are their RSDs. The RSD for concentration was computed from the last two columns of Table 3-2. The RSDs for the densities are from Table 2-1. The RSDs for volumes are from Yorgesen (2000).

The eighth column of the table is a repeat of the inventory column from Table 3-1. Column nine lists the RSDs of the inventories. This column is the square root of the sum of the three individual RSDs squared; i.e., it is given by Equation (2.4). Rows seven and eleven give the inventory estimates for the solid and liquid portions of the waste. The standard deviation of the inventory, SD(I), was computed using Equation (2.12). The RSD(I) was derived from SD(I) and I. Likewise, all of the inventory estimates from the different phases were combined into a total inventory and a standard deviation of the total using Equation 2-12. The estimate of the Al inventory is 47972 kg and an estimate of one standard deviation is 17844 kg, or one RSD is about 37%.

Table 3-1. Example Calculations, Aluminum in Tank 241-U-109

Waste Phase	Waste Type	Inventory	Concentration	Inventory Calculation
Units		Kg	µg/g	
Sludge	CWR1sludge(solids)	22100	170000	1.71g/mL*76kL*170000 µg/g*0.001*1
Saltcake	S1saltcake(liquid)	5870	19864	1.47g/mL*201kL*19863.9µg/g*0.001*1
Saltcake	S1saltcake(solids)	10700	12800	1.67g/mL*500kL*12800µg/g*0.001*1
Sludge	S1saltcake(liquid)	321	19864	1.47g/mL*11kL*19863.9µg/g*0.001*1
Saltcake	S2Saltslurry(solids)	8980	12800	1.67g/mL*420kL*12800µg/g*0.001*1

Notes:

CWR1 sludge =Reduction oxidation (REDOX) wastes, 1952-60, aluminum clad fuel
S1saltcake = 242-S Evaporator campaign, 1973-6, S-102 feed
S2saltslurry =242-S Evaporator campaign 1977-80, SY-102 feed

Table 3-2. Summary Statistics for Aluminum in Tank 241-U-109

Waste Phase	Volume	Density	Units	Mean Concentration	SD(Mean)
Units	KL				
Sludge(solid)	76	1.71	µg/g	170000	28536
Saltcake(liquid)	201	1.47	µg/mL	29200	100
Saltcake(solid)	500	1.67	µg/g	12800	579
Sludge(liquid)	11	1.47	µg/mL	29200	100
Saltcake(solid)	420	1.67	µg/g	12800	579

Table 3-3. Example Calculations for Combining Relative Standard Deviation (RSDs) and Standard Deviation (SDs)

Waste Phase	Concentration	RSD(C[1])	Density	RSD(D[2])	Volume	RSD(V[3])	Inventory	RSD(I[4])	SD(I)	Comment
Units	µg/g		g/mL		kL		kg		kg	
Solid Waste				Table 2-1				Eqn 2.4	Eqn 2.5	
Sludge	170000	0.1679	1.71	0.0755	76	0.6757	22100	0.7003	15476	
Saltcake	12800	0.0452	1.67	0.0755	500	0.0213	10700	0.0905	969	
Saltcake	12800	0.0452	1.67	0.0755	420	0.0213	8980	0.0905	813	
Total Solid							41780	0.4131	17258	Eqn 2.12
Liquid Waste	µg/mL									
Saltcake	29200	0.0034	1.47	0.059	201	0.0213	5870	0.0628	369	
Sludge	29200	0.0034	1.47	0.059	11	0.6757	321	0.6782	218	
Total Liquid							6191	0.0947	587	Eqn 2.12
Total							47972	0.3720	17844	Eqn 2.12

Notes:

[1]C: Concentration estimate
[2]D: Density estimate
[3]V: Volume estimate
[4]I: Inventory estimate µg/mL

4.0 REFERENCES

Ferryman, T. A., B. G. Amidan, G. Chen, S. A. Hartley, C. A. LoPresti, J. G. Hill, T. J. DeForest, F. Gao, K. M. Remund, and B. C. Simpson, 1998, *Summary of Uncertainty Estimation Results for Hanford Tank Chemical and Radionuclide Inventories*, PNNL-12003, Pacific Northwest National Laboratory, Richland, Washington.

Hogg, R. V., and A. T. Craig, 1971, *Introduction to Mathematical Statistics*, third edition, The Macmillan Company, New York, New York.

S-PLUS 2000, 2000, *S-PLUS 2000*, Data Analysis Product Division, MathSoft, Inc., Seattle, Washington

Yorgesen, J. B., 2000, *Uncertainty Estimates of Tank Volumes*, RPP-6570, COGEMA Engineering Corp., Richland, Washington.

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APPENDIX A

REVIEW CHECKLIST

REVIEW CHECKLIST

Document Reviewed:
Statistical Methods for Estimating the Uncertainty in the Best Basis Inventories by S. R. Wilmarth and L. Jensen, RPP-6924

Scope of Review:

Yes	No	NA		
○	○	⊗	*	Previous reviews complete and cover analysis, up to scope of this review, with no gaps.
⊗	○	○		Problem completely defined.
○	○	⊗		Accident scenarios developed in a clear and logical manner.
⊗	○	○		Necessary assumptions explicitly stated and supported.
○	○	⊗		Computer codes and data files documented.
○	○	⊗		Data used in calculations explicitly stated in document.
○	○	⊗		Data checked for consistency with original source information as applicable.
⊗	○	○		Mathematical derivation checked including dimensional consistency of results.
⊗	○	○		Models appropriate and used within range of validity or use outside range of established validity justified.
○	●	⊗ (DAR)		Hand calculations checked for errors. Spreadsheet results should be treated exactly the same as hand calculations.
○	○	⊗		Software input correct and consistent with document reviewed.
○	○	⊗		Software output consistent with input and with results reported in document reviewed.
○	○	⊗		Limits/criteria/guidelines applied to analysis results are appropriate and referenced. Limits/criteria/guidelines checked against references.
○	○	⊗		Safety margins consistent with good engineering practices.
⊗	○	○		Conclusions consistent with analytical results and applicable limits.
⊗	○	○		Results and conclusions address all points required in the problem statement.
○	○	⊗		Format consistent with appropriate NRC Regulatory Guide or other standards.
○	○	⊗	*	Review calculations, comments, and/or notes are attached.
⊗	○	○		**Document approved.**

Daniel A. Reynolds, D A Reynolds — 8/31/00

Reviewer (Printed Name and Signature) — Date

*Any calculations, comments, or notes generated as part of this review should be signed, dated and attached to this checklist. Such material should be labeled and recorded in such a manner as to be intelligible to a technically qualified third party.

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