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An Approximate-Reasoning-Based Method for Screening Flammable Gas Tanks

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EXECUTIVE SUMMARY

High-level waste (HLW) produces flammable gases as a result of radiolysis and thermal decomposition of organics. Under certain conditions, these gases can accumulate within the waste for extended periods and then be released quickly into the dome space of the storage tank. As part of the effort to reduce the safety concerns associated with flammable gas in HLW tanks at Hanford, a flammable gas watch list (FGWL) has been established. Inclusion on the FGWL is based on criteria intended to measure the risk associated with the presence of flammable gas. It is important that all high-risk tanks be identified with high confidence so that they may be controlled. Conversely, to minimize operational complexity, the number of tanks on the watchlist should be reduced as near to the "true" number of flammable risk tanks as the current state of knowledge will support.

The FGWL screening process has several functional steps. The first is to determine if the available information is sufficient to allow a meaningful evaluation of the tank. If so, the evaluation is performed. The evaluation results in a recommendation on whether the tank should be on the watch list. In this latter step, some statement of the confidence associated with the recommendation is required. The actual process of going from some universe of information for a tank to a clear recommendation on tank classification is a complex, frequently implicit combination of inferences about flammable gas phenomenology. These inferences about gas generation, composition, retention, and release characteristics for a tank are drawn from a large, diverse, uncertain, and often contradictory universe of information. This universe includes

- observations associated with gas-release events (GREs);
- measurements and associated models for predicting volumes of retained gas; and
- waste properties associated with empirical models to roughly estimate the potential for gas generation, retention, and release.

It is quite common for a conclusion drawn from one set of data and models to be diametrically opposed by some other set of data and models.

This report presents an alternative to existing approaches for FGWL screening based on the theory of approximate reasoning (AR) (Zadeh 1976). Our AR-based model emulates the inference process used by an expert when asked to make an evaluation. The FGWL model described here was exercised by performing two evaluations.

1. A complete tank evaluation where the entire algorithm is used. This was done for two tanks, U-106 and AW-104. U-106 is a single shell tank with large sludge and saltcake layers. AW-104 is a double shell tank with over one million gallons of supernate. Both of these tanks had failed the screening performed by Hodgson et al.
2. Partial evaluations using a submodule for the predictor likelihood for all of the tanks on the FGWL that had been flagged previously by Whitney (1995).

The first evaluation provides insight into issues concerning input data, computational effort and interpretation of the results. In the second evaluation, results from the AR model were compared with those from the Whitney model.

One significant difference between the AR model results and those of Hodgson et al. for the complete tank evaluations involved the interpretation of the long-term level data. Both of the tanks failed the Hodgson screen based on this model for retained gas volume. However, in the AR model, the

quality associated with the data was inferred to be "poor" because of the large influence of the correction terms.

In the Whitney analysis, the correlation between barometric pressure and waste level was examined. If a large amount of gas is present, then with several important qualifications, there should be a strong, negative linear correlation between pressure and level. Whitney examined a large number of tanks (not on the current FGWL) and found 37 tanks where the correlation was found to be strong for at least one level sensor. These tanks also were examined using the AR model. In addition to the threshold criterion used by Whitney, additional statistical measures and judgments about data quality were used. Rulebases also were added to take into account relative instrument quality and to resolve differences between the inferences drawn for individual sensors. At the 0.95 quantile, the AR model classified 11 of these tanks as having a strong correlation and inferred that for two of the tanks the correlation was weak. Of the remaining 24 tanks that were classified as unresolved, an additional classification could be made. By observing the cumulative distribution function (CDF) for the output likelihood, it was possible to differentiate between tanks where the data were of reasonable quality but contradictory and tanks where the data were judged too poor to allow a definitive judgment to be made. Eleven tanks fell in this later category. The capability to make these types of judgments explicit is an important attribute of approximate reasoning.

Screening waste tanks for flammable gas is a difficult undertaking. The difficulty arises because of the incomplete understanding of the relevant phenomenology and the need to use partial, and apparently contradictory, data in models that are themselves incomplete. Our pilot study of the application of the AR methodology to this problem is encouraging. The inductive logic structure and the associated series of implication rule bases make a realistic representation of the current state of knowledge possible. The use of linguistic variables and fuzzy sets provides a way to combine qualitative and quantitative data in a consistent way. The combination of fuzzy and probabilistic approaches in the same model allows for a natural treatment of both uncertainty and ambiguity.

The pilot model showed that the effort required to build an AR model for a relatively complex problem is reasonable and that computational requirements are acceptable. Preliminary analyses with the model clearly demonstrated the value of incorporating qualitative judgments about data and models directly into the screening logic. Differences between the results obtained with the AR model and those obtained previously often could be explained as a consequence of the more detailed inferences about model and data validity included in the rule bases. We conclude that AR is a promising tool for this type of screening problem and that further development in this area would be useful.

Nomenclature

χ	Temporal correlation for short term level and temperature changes
CDF	Cumulative probability distribution function
C_M	Maximum dome space flammable gas concentration using WHC quick screen
C_O	Total organic carbon
Δh	Long-term waste level change
δh	Short-term waste level change
$\delta \theta$	Short-term waste temperature change
ϕ	Waste porosity
F_{li}	Liquid fraction of waste type i
γ	Degree of membership vector
G_i	Gas generation potential for ith condition
h	Waste level
h'	Current waste level
I_i	Number of level measurement intervals for level sensor i
L	Waste level
λ	Centroid associated with degree of membership vector
L_i	Gas likelihood for i^{th} event or condition
$\mu(A)$	Membership function for fuzzy set A
$\mu(A,x)$	Value of membership function for fuzzy set A at x
N_i	Multiplicity of measurements for dome space parameter i
PDF	Probability density function
P_{si}	Probability of random occurrence of negative interval proportion for level sensor i
Q	Quality judgment for long-term level change correction terms
q_i	i^{th} quantile of a cumulative probability distribution function
q'''	Volumetric heat generation rate
R^2_i	Linear mean square error coefficient for level sensor i
R_i	Gas retention potential for i^{th} condition
S	Waste specific gravity
S_i	Level-barometric pressure correlation slope for level sensor i
T	Waste temperature
V_i	Volume of waste type i
X_i	Certainty of estimated dome space maximum value for parameter i

Subscript Nomenclature

81	Level measured in 1981
B	Barometric pressure correlation
C	Combined
D	Dome space
D	Derivative
Δh	Waste level change
E	Evaporation
E	Enabler
e	ENRAF level instrument
f	Food Instrument Corporation level instrument
F	Aggregate
g	Dome space gas concentration
h	Waste level
I	Indicator
L	Liquid
m	Manual tape level instrument
m	Measured
N	Supernate
n	Neutron log level instrument
O	Dome space overpressure
P	Predictor
P81	Level change estimate for years before 1981
S	Solids
SC	Salt cake
T	Total
C	Salt cake
D	Sludge
SW	Wet salt cake
I	Interstitial

AN APPROXIMATE-REASONING-BASED METHOD FOR SCREENING FLAMMABLE GAS TANKS

1.0. INTRODUCTION

In the interest of safety, a flammable gas watch list (FGWL) has been established for high-level waste tanks at the Hanford Site. Inclusion on the FGWL is based on criteria intended to measure the risk associated with the presence of flammable gas. Such tanks receive increased Department of Energy (DOE) oversight and may be governed by special administrative controls that are designed to reduce the risk from flammable gas ignition accidents. These measures have a substantial effect on tank farm operations, and thus, selection of watchlist tanks is doubly constrained. It is important that all high-risk tanks be identified with high confidence so that they can be controlled. Conversely, to minimize operational complexity, the number of tanks on the watchlist should be reduced as near to the "true" number of flammable risk tanks as the current state of knowledge will support.

There are several steps in the FGWL screening process. The first is to determine if the available information is sufficient to allow a meaningful evaluation of the tank. If so, the evaluation is performed. The result of the evaluation is a recommendation on whether the tank should be on the watchlist. In this latter step, a statement of the confidence associated with the recommendation is required. The actual process of going from some universe of information for a tank to a clear recommendation on tank classification is a complex, frequently implicit combination of inferences about flammable gas phenomenology. These inferences about gas generation, composition, retention, and release characteristics for a tank are drawn from a large, diverse, uncertain, and often contradictory universe of information that includes the following.

- Observations associated with gas-release events (GREs) such as level, temperature, and pressure fluctuations as well as dome-space gas concentrations.
- Measurements and associated models for predicting volumes of retained gas. These include long-term level change and the use of the correlation between level fluctuations and barometric pressure as well as the retained gas sampler and voidmeter sensors.
- Waste properties associated with empirical models to roughly estimate the potential for gas generation, retention, and release. These include waste specific gravity, total organic carbon, specific activity, and waste stratigraphy and strength.

It is quite common for a conclusion drawn from one set of data and models to be diametrically opposed by some other set of data and models. Data vary in terms of quality and the degree of associated uncertainty, and models have varying powers of prediction. As a result, the evaluation must contend with an entire series of qualitative judgments about what inferences regarding flammable gas phenomenology are possible and how to resolve discrepancies among them. Compounding this problem is the fact that perhaps as many as 177 tanks must undergo the screening, and some demonstration of consistent evaluation is needed. This is difficult to achieve because of the wide variations in waste type, the large differences in installed instrumentation, and the differences in the historical data base.

These attributes of the FGWL problem define a number of features that would be desirable in any screening methodology, including the following.

- The method would have the ability to use both quantitative and qualitative data to draw inferences.
- All of the relevant information on phenomenology would contribute to the final inference about the flammable gas state of a tank.
- An explicit evaluation of relative importance would be associated with a particular data/model set.

- The uncertainty associated with both inherent variability in data and incomplete state of knowledge incorporated into phenomenological models would be represented.
- There would be a clear description of the flammable gas state of a tank and the associated uncertainty.
- Tanks would be treated consistently to ensure comparability of evaluations.
- Tank evaluations would be traceable and repeatable.
- The approach would be compatible with the detailed level of review associated with the FGWL.

In this report, we propose an alternative to existing approaches for FGWL screening based on the theory of approximate reasoning (AR) (Zadeh 1976). Our AR-based model emulates the inference process used by an expert when asked to make an evaluation. The primary characteristics of the AR model developed here are listed below.

- The relationship between data and models is defined explicitly by an inductive logic structure.
- The inference process is implemented using formal logical implication.
- All of the relevant data are transformed into natural language expressions called linguistic variables. A linguistic variable is represented in the model using fuzzy sets. The degrees of membership in fuzzy sets are used directly in the logical implication operation.
- Judgments about the importance or quality of data and models also are treated as linguistic variables. These judgments are incorporated explicitly into the inference model.
- Uncertainty is represented using standard Monte Carlo techniques for random variables. Ambiguity is treated using fuzzy sets.

We explain how the characteristics listed here are incorporated into an AR-based model for FGWL screening. Additionally, it will be shown that this method can be used to ensure that evaluation criteria are applied consistently to all tanks. The task of documenting and defending the evaluations is facilitated by the fact that the logic structure and the sequence of inferences in the AR model can be traced directly to the subject matter experts used in building the model. That is, the evaluation framework is developed principally by the experts rather than created *a priori*.

Section 2 is an overview of the FGWL history and the current approach to screening followed by an overview of the AR methodology. Here we discuss the basic concepts used to construct an inductive logic model that defines the relationships between data and models. The utility of the fuzzy set representation of linguistic variables is discussed, and the use of rule bases composed of sets of logical implications to carry out a series of forward-chaining inferences is explained. The evaluation process in an AR model is examined using one small segment of the complete pilot FGWL AR model. We develop the inductive model for making inferences about retained gas from the long-term level change data and show how a likelihood statement about retained gas is obtained.

The development of the AR model is described in Sec. 3. We consider the form in which the evaluation result is to be expressed and present an inductive logic structure that has this output. The logic structure defines the relationships between the data, models, and judgments considered to be relevant to the screening process and defines the order in which the sequence of required inferences is to occur. The inferences are carried out using rule bases that specify the implications that can be drawn from related elements of evidence. Example implication rule bases for each relation in the logic structure are presented. These are intended to illustrate the types of judgments that can be made and the degree of sophistication that can be incorporated into the inferences. We develop the natural language descriptions for elements of evidence and show how membership in fuzzy sets allows us to transform quantitative data into linguistic variables and why these variables are used in the inference rule bases. It is important to note that the emphasis in this report is on the AR model and the issues associated with applying it to the FGWL screening problem. The actual logic structure, rule bases, and membership functions given in this report are intended to illustrate the concepts and show the utility of the AR method for this problem.

The implementation of the AR model is discussed in Sec. 4. We describe the computer program used to evaluate the logic structure and the preparation of the input data. The approach used for the Monte Carlo simulation to obtain statistics for both the intermediate and final likelihoods and the use of probability distributions to describe the uncertainty associated with both the input and the final output are discussed.

Results from several evaluation simulations are given in Sec. 4. We provide complete evaluations for two tanks, U-106 and AW-104. U-106 is a single-shell tank containing thick layers of salt cake and sludge, and AW-104 is a double-shell tank with over a million gallons of supernate. The entire inference chain is examined, and the interpretation of the output is discussed. We also performed simulations using just the barometric pressure logic module for all of the tanks on the FGWL and those that failed the screening described in Hodgson (1995). The results obtained using the AR approach are compared with both of these lists, and the differences are explained in terms of the features of the AR model. Again, we emphasize that the model here has been developed for method testing and to provide illustrative results. Although an attempt has been made to provide reasonable characteristics for the model factors, the results must be considered preliminary. Additional expert judgment is required to refine the screening logic so that it represents the current state of knowledge about flammable gas phenomenology.

In Sec. 6, we review the development of the AR model and discuss the advantages of this approach based on the results of our pilot model testing. We conclude that the AR model is a powerful analytical approach to FGWL screening and that a full-scale implementation is practical. We outline briefly the steps needed to build a complete model for FGWL screening.

2.0. OVERVIEW OF FLAMMABLE GAS WATCH LIST SCREENING METHODOLOGIES

In this section, we consider the options available for FGWL screening. To do so, we review the history of the watchlist and the development of the criteria currently used to screen tanks. The methodology currently used to implement these criteria is discussed, and known problems with this approach are noted.

We also introduce the concepts of AR. These include a discussion of information and evidence and how these data are processed using a formal logic structure. The concept of linguistic variables and fuzzy sets is introduced using a simple nontechnical example. We then illustrate the use of an AR model for screening using a small logic module associated with long-term level rise and show how inferences are drawn. This sets the stage for the detailed development of the AR model in Sec. 3.

2.1. Flammable Gas Watch List History

A useful starting point in the description of the screening methodology is to examine the evolution of the FGWL.

The original watchlist was established in 1990. The tanks initially on the watchlist were those believed to pose a potential explosion risk from the reaction of ferrocyanide/nitrate mixtures. This list was compiled in response to the General Accounting Office and to the exposure given to the problem during the confirmation hearings in the Senate for the Defense Nuclear Facility Safety Board. Later in 1990, the list was extended to include, first, tanks with a flammable gas hazard and, shortly thereafter, those with organic and high-heat issues. Criticality issues were added to the watchlist concerns still later. Tanks on the original FGWL were determined using a "slurry growth" criterion based on a cursory review of tank records and supporting documents. The original FGWL had 20 tanks (WHC 1990).

The declaration of a flammable gas unreviewed safety question (USQ) was made in April 1990 by the Department of Energy/Richland Operations Office (DOE/RL) (Lawrence 1990). Although the primary emphasis was on Tank 241-SY-101, the scope covered any tank that had the proper flammable gas generation and retention/release behavior.

Public Law 101-510 was signed late in 1990. Section 3137, the Wyden amendment, specifically addresses significant safety issues in waste tanks at Hanford. Section 3137 requires that the Secretary of Energy "identify . . . (which) tanks at Hanford . . . may have a serious potential for release of high-level nuclear waste due to uncontrolled increases in temperature or pressure." It places restrictions on waste transfers to these tanks and mandates that the DOE pay special attention to their safety. The watchlist tanks in each category were identified as meeting the criteria to be classified as "Wyden amendment" tanks. That is, *watchlist* and *Wyden amendment* became synonymous.

At approximately the same time, an effort was begun to develop more detailed criteria for the FGWL (Harmon 1991). A ranking system was developed to evaluate all waste tanks in terms of two conditions.

1. Potential for flammable gas production
2. Potential for gas retention and release at concentrations above the lower flammability limit (LFL)

Every tank was ranked for both conditions using a number of weighted factors, including surface-level fluctuations and the detected presence of flammable gas. No attempt was made to incorporate specific models for gas release. With this method, all of the original FGWL tanks were confirmed to be on the watchlist, two tanks were added to the list, and one additional tank (SX-109) was included because it shared a ventilation system with a number of FGWL tanks for a total of 23 tanks. Two more tanks were identified as having the potential for gas retention (AW-101 and U-107) in 1993 and 1994. The tanks currently on the FGWL are listed in Table 2-1.

Table 2-1
Tanks on the Flammable Gas Watch List

A101	S112	SY103
AN103	SX101	T110
AN104	SX102	U103
AN105	SX103	U105
AW101	SX104	U107
AX101	SX105	U108
AX103	SX106	U109
S102	SX109	
S111	SY101	

Inadequacies in the FGWL criteria were well-known, and Westinghouse Hanford Company (WHC) proposed new criteria in 1994 (Hopkins 1994). The new criteria explicitly addressed the phenomenology associated with gas release and the subsequent release of radioactive material to the environment. In addition, the Hopkins report (1994) examines the relationship of the criteria with the Wyden amendment. It was proposed that the phrase "serious potential for release" be interpreted in terms of the WHC risk acceptance guidelines (RAGs) for co-located worker and offsite consequence. In this context, the provision is interpreted to mean "identify tanks with a *credible* potential for *serious* release" (emphasis in original). The criteria are given in Table 2-2. A safety factor of 4 is used for all cases. Basically, this means that either the flammable gas concentration must be below 25% of the LFL or any over-pressurization must be less than 25% of that which could cause a serious release. This factor is based on common standards for fire safety (NFPA 1995) and pressure vessel design. The National Fire Protection Association limits normally are associated with a control for operations in a flammable environment. Note that the probability associated with exceeding these levels is not defined.

Table 2-2
Westinghouse Hanford Company Criteria for Flammable Gas Tanks (Hopkins 1994)

Condition in Tank Dome Space or Ventilation Header	Criterion
Flammable gas at uniform concentration—steady-state condition. Flammable gas at uniform concentration—dispersed-release condition	The tank could have a uniform flammable gas concentration greater than 25% of the LFL in the dome space or ventilation headers
Flammable gas concentrated in a region (plume)	The tank could release a plume [in-progress episodic gas release (EGR)] with a volume greater than 25% of that plume volume that, if ignited, could explode and cause a serious release to the environment. (The volume varies from tank to tank. It is expected to be about 0.25% of the dome-space volume.)
Over-pressurization	The tank could have an over-pressure of more than 25% of the over-pressure that could cause a serious release to the environment. (The over-pressure varies from tank to tank. An over-pressure of about 10 in w.g. could cause a serious release.)

Because of the Wyden amendment, responsibility for the FGWL resides with the Secretary of Energy, who has delegated it to the Assistant Secretary for Environmental Management. No DOE Orders define how the watchlist is to be modified, that is, how tanks are to be added or removed (Lytle 1994). In practice, tanks are added on the advice of WHC and removed on a tank-by-tank basis. To date, no tank has been removed from the FGWL, although it is expected that Tank SY-101 will be removed in the near future.

An effort has been under way since 1995 to evaluate all of the waste tanks against the 1994 criteria (Hopkins 1995; Hodgson 1995). In this work, models are used to relate tank observables, for example, waste level fluctuations, to possible dome-space gas concentrations. In conjunction with this work, an additional method has been developed to detect the presence of *in situ* gas. During the Tank SY-101 mixer pump project, it was noted that changes in waste level should be correlated with barometric pressure because of the large amount of retained gas in the waste. Pacific Northwest National Laboratory (PNNL) has refined this technique and applied it to all of the tanks (Whitney 1995). The results suggest that gas may be retained in 38 additional tanks. Using further assumptions, WHC has calculated that 25 of these tanks would exceed the1994 criteria (>25% LFL steady state or from GREs) (Bacon 1996). These tanks are listed in Table 2-3. No decision has been made by DOE/RL as to whether these additional tanks are to be on the FGWL.*

2.2. Current Screening Methodology

Because of the statutory considerations associated with the Wyden amendment and the operational concerns noted above, the methodology used in Hodgson (1995) has received close scrutiny. Problems noted during the various reviews were considered sufficiently serious to warrant studying the development of an alternative screening procedure for FGWL tanks. To understand the basis for the methodology described below, it is necessary to briefly discuss the criticisms associated with the existing screening techniques.

Figure 2-1 is a simplified schematic of the logic structure used in Hodgson. Here, three estimates of retained gas volume are made using tank level discrepancy, $V_g(\Delta h)$; the correlation between level and barometric pressure perturbations, $V_g(B)$; and a quick screen method, $V_g(Q)$ proposed in Hopkins (1995).** For $V_g(B)$, the maximum value calculated from the available level sensors is used. That is, the maximum slope of level change vs pressure obtained from one or more of the FIC, ENRAF, manual tape, and neutron log level sensors is converted to $V_g(B)$ using a simple gas-spring model. This maximum is

Table 2-3
Preliminary Recommendation by Barton for Flammable Gas Watch List Membership

A103	BY109	TX102
AW104	C104	TX111
AY101	C107	TX112
BX107	S101	TX113
BY101	S103	TX115
BY102	S105	U102
BY103	S106	U106
BY105	S107	
BY106	S109	

*More recently another group was assembled by DOE/RL to review the recommendations in Bacon. This group, known as the Vieth Committee, also has issued a report with its own recommendations on which tanks should be on the FGWL.

** Each of these approaches to calculating gas inventory will be discussed in detail in Sec. 3.

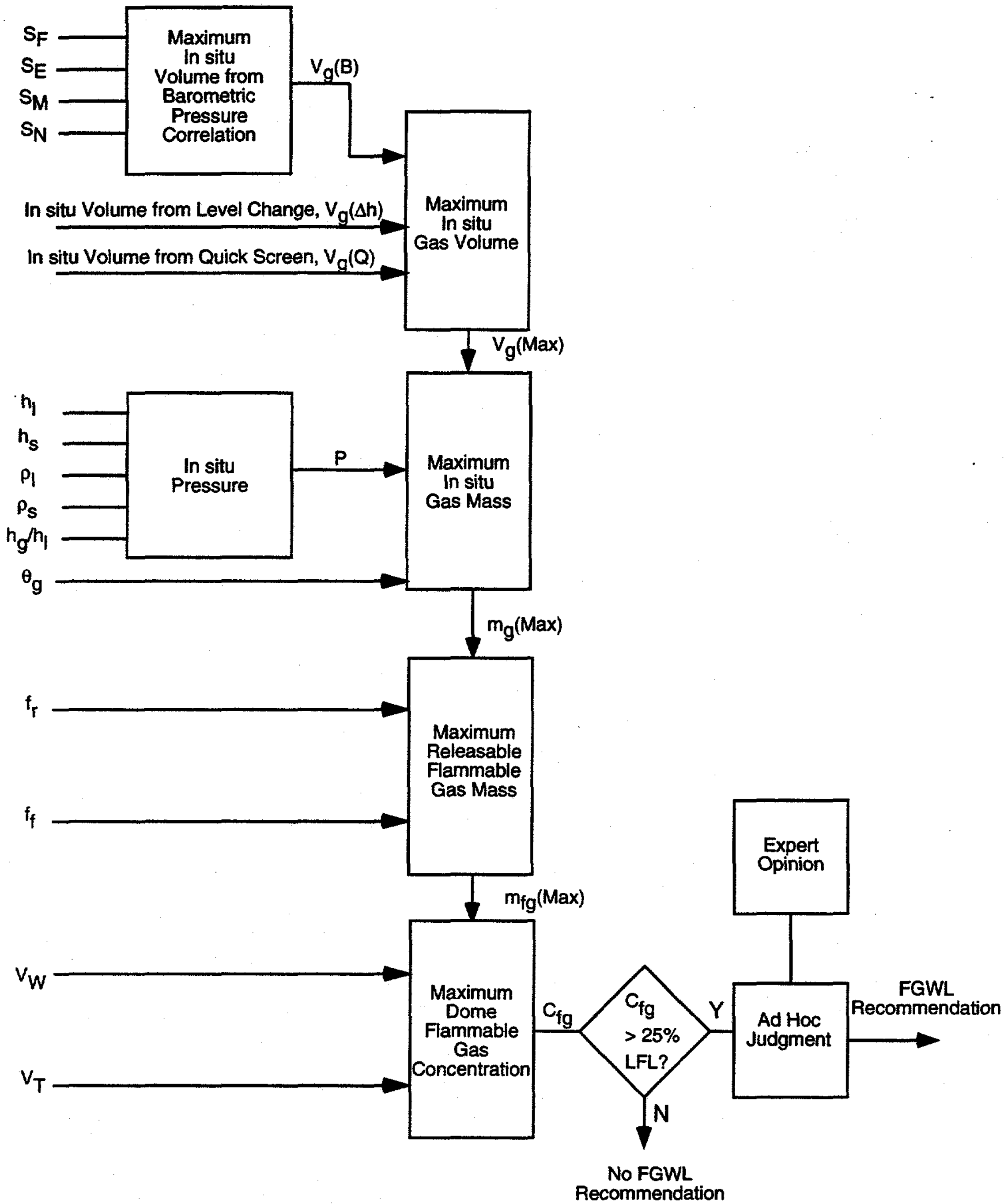

Fig. 2-1. Logic schematic for screening approach in Hodgson.

compared with $V_g(\Delta h)$ and $V_g(Q)$. Again, the maximum is selected. Additional parameters are introduced to calculate the mass of retained gas and the amount released into the dome space assuming the gas can be mobilized. By assuming a gas composition, this mass can be related to a dome-space flammable gas concentration, C_{fg}. The maximum value for C_{fg}, $C_{fg}(max)$ is compared with a threshold value, $C_{fg}(t)$, the previously mentioned 25% of LFL. If $C_{fg}(max)$ exceeds this threshold, the tank is considered to have failed the screen and is a candidate for the FGWL. Tank observables and parameters are characterized with probability distributions so that Monte Carlo simulation can be used to provide some measure of uncertainty as input for the final decision. However, only point estimates are used in the decision process.

The preliminary comparison with the LFL obtained in this manner is not the final screening result. An additional evaluation (referred to here as the *ad hoc* judgment) follows and is made for two reasons.

- Calculation of C_{fg} (max) does not use all of the information available about gas generation, retention, and release phenomenology in a tank.
- Practically speaking, too many tanks had a value of C_{fg} (max) above the threshold criterion.

In the *ad hoc* step, other information or expert judgment is incorporated, and a final evaluation judgment is made. Thus, much of the decision process is done "off line" but is nevertheless an essential element in the screen being used. The logic structure used in this evaluation is reasonable, but difficulties are experienced in practice. Note that although uncertainty in observational or parametric data can be propagated when determining C_{fg} (max), there is no consistent method applied to represent uncertainty in expert judgment. However, the relative quality of sensors or calculational models represents the major sources of uncertainty that must be evaluated by the experts during the screening process. Second, it can be seen that in this logic structure, the expert judgment and the value of the threshold for the criterion, C_{fg} (t) = 0.25 LFL, are not independent. That is, the decision in the *ad hoc* step is affected by the conservatism of the criterion.* This makes it extremely difficult to provide best-estimate/degree-of-conservatism comparisons. Finally, with the structure discussed here, it is difficult to ensure that the screening is consistent, and this poses problems during review.

2.3. Overview of an Approximate-Reasoning Based Screening Method

The considerations above suggest that the design and implementation of a new screening method should be based on a formalism that is both robust and adaptable and in which all of the necessary judgments are defined explicitly. The theory of approximate reasoning provides such a formalism.

2.3.1. Information, Evidence, and Uncertainty. The general structure underlying the AR method developed in this report is shown in Fig. 2-2. We begin with some universe of information about a tank to be screened. The universe consists of both qualitative and quantitative data. This information is not necessarily in a form in which it is directly useful, and therefore, some processing of the data is required. We denote this processed data as a body of evidence, and only elements within it will be considered in the screening process. Elements of evidence must be related to each other in some meaningful manner. This is done by way of formal structures with logical operations relating the evidence to produce a series of forward-chaining inferences. The output from the logic structure is a description of the system called a "state vector."

The state vector is a concise description of a system, in this case, the tank undergoing the screening. The elements of a state vector are always assumed to include some component of uncertainty that reflects imprecision or ambiguity in the knowledge of the system state. Finally, the system state vector is used in a decision model where some definite statement about the system is made. Note that the level of abstraction increases as we move through the process. We now consider each step in this sequence in more detail.

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*In this structure, the expert considers the validity of the calculated C_{fg} (max) in relation to the value of C_{fg} (t). Thus, the judgment incorporates aspects of uncertainty and degree of conservatism simultaneously in a way that is impossible to quantify.

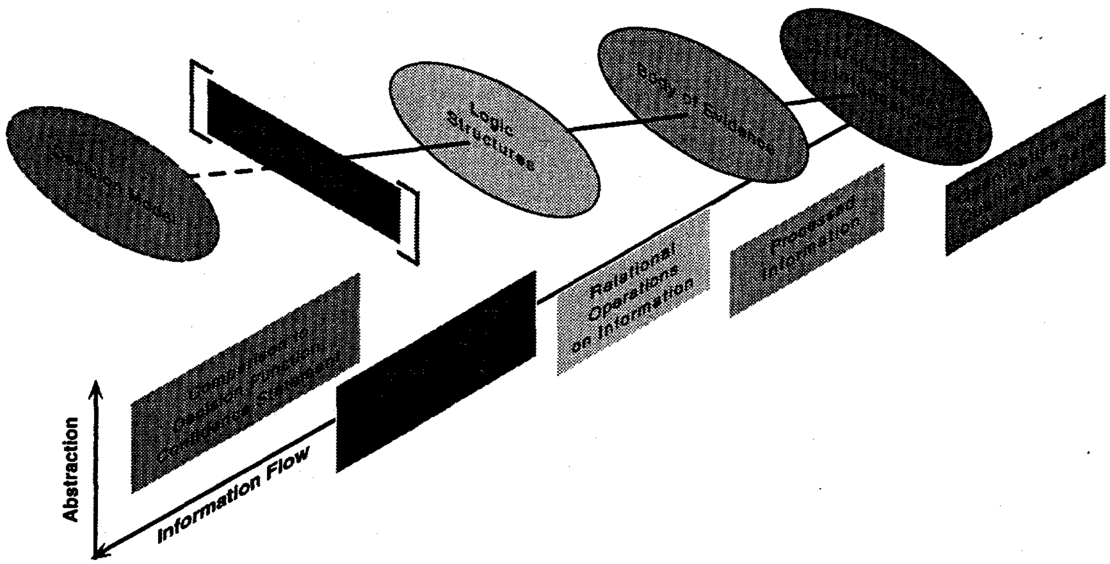

Fig. 2-2. Overall structure for the AR model.

The relationship between the universe of information and a body of evidence is shown in Fig. 2-3. The primary data consist of what can be considered "raw" information about the tank. These include gas concentration measurements, waste characteristics, and level sensor data. Information processing is needed to make these data useful. Processing involves phenomenological models, detailed numerical simulation, and specific expert judgment. These operations place a piece of information in a useful context. Information of this sort forms a body of evidence.

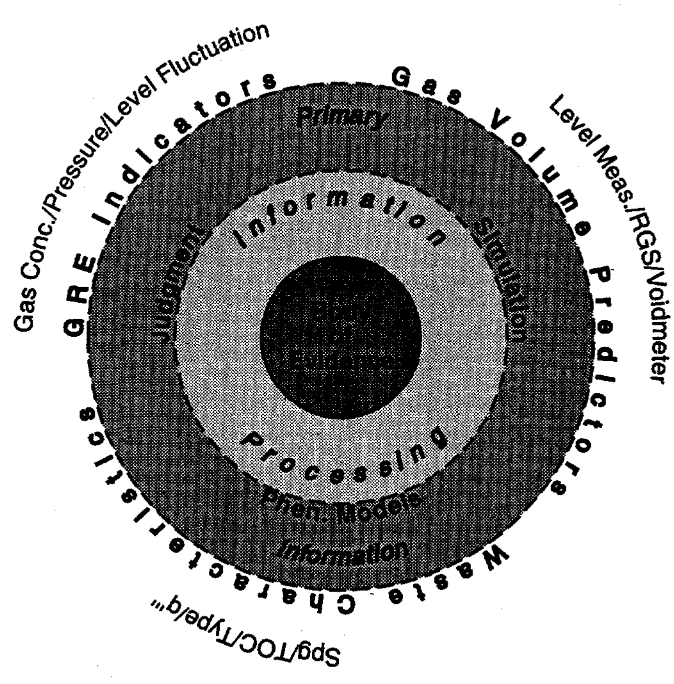

Fig. 2-3. Conceptual relationship between the universe of information and a body of evidence for the FGWL problem.

In an AR model, the elements of evidence are handled as linguistic variables; that is, natural language descriptors are used. For example, we can characterize the temperature in a room as "too cold," "comfortable," or "too hot" without actually measuring the temperature. The descriptors are used to define sets in which the variable of interest, in this case the temperature in the room, may belong. The sets used in AR are fuzzy—a variable may belong to sets that traditionally might be considered to be mutually exclusive.

For our example, the temperature could be said to belong to all the fuzzy sets {Too Cold}, {Comfortable}, and {Too Hot}. Membership in a fuzzy set can vary between 0 and 1, with 1 implying full membership and 0 implying nonmembership. For quantitative elements of evidence, the degree of membership (DOM) in a set is assigned using membership functions. The numerical value of the degree of membership in a set S_j is determined by $\gamma(x, S_j) = \mu(x,S_j)$, where $\mu(x,S_j)$ is the actual membership function. Possible membership functions for the room temperature are shown in Fig. 2-4. If the temperature in the room is 70°F, this results in DOMs of 0.5 in {Too Cold} and {Too Hot} and a DOM of 1.0 in {Comfortable}. We denote the three DOMs in these sets by the vector $\gamma_T = \{0.5,1,0.5\}$; membership in {Comfortable}, for example, is $\gamma(T, \text{Comfortable}) = 1.0$. It is important to note that a small change in temperature will have a similarly small effect on the degrees of membership in the sets.

The more traditional approach is to use threshold values to define the sets. With this approach, a particular temperature can only belong to one set. Such sets are referred to as crisp, and a small change in temperature could completely change the set to which it belongs. So far, we have been dealing with quantitative measures for temperature. However, we might chose instead to use our subjective judgment to qualitatively define the room temperature. The degrees of membership in the sets then can be assigned directly. For example, the judgment "a little too cold" could be converted directly to a DOM of {0.3,0.7,0} without explicit membership functions. Note that when using either quantitative or qualitative temperature measures, the use of fuzzy sets allows for ambiguity in classification of the temperature.

An element of information can be either quantitative or qualitative, but it is important to note that in either case, it is almost inevitably uncertain. If an element is defined numerically, it is treated as a classic random variable characterized by a probability density function. Definition of the parameters in the density function then characterizes the uncertainty. A qualitative element is always considered to be a linguistic variable. These also can be random variables.

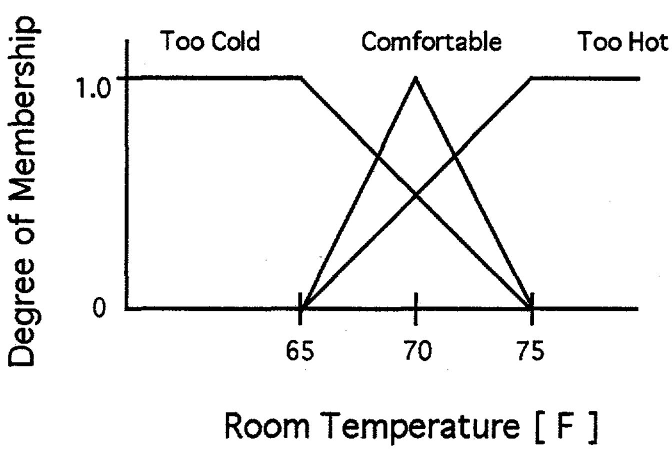

Fig. 2-4. Membership functions for fuzzy sets used to describe room temperature.

The total uncertainty associated with an element of evidence is composed of two components—aleatory and epistemic—as shown in Fig. 2-5. The aleatory component represents the inherent variability in a parameter. Processes such as radioactive decay and turbulence exhibit aleatory uncertainty. They also are commonly referred to as stochastic processes. The epistemic component represents state-of-knowledge uncertainty. For example, the assumptions and approximations made in a model induce epistemic uncertainty in the results. That is, there is some doubt about how well the model represents physical reality. It is important to note that epistemic uncertainty is often greater than the aleatory component in many problems.

2.3.2. Organizing Evidence Using a Logic Structure. The connection between the elements in the body of evidence and a logic structure is shown in Fig. 2-6. The logic structure defines a set of relationships between the elements of evidence. The nature of the individual branch junctions depends on the particular type of relation used. A relation is a general function that maps multiple inputs into a single output. In this report, we consider only binary relations. Many different types of relations, both numerical and logical, are possible. However, in an AR model, the only relation used is formal logical implication. We refer to this as an implication junction. The implications are of the form "If A and B then C," or "A and B implies C," written symbolically as

$$(A \wedge B) \rightarrow C\,.$$

For example,

Implication 1: If the barometric pressure correlation is *good* and the unexplained level change is *large*, then the presence of a significant quantity of retained gas is *quite likely*.

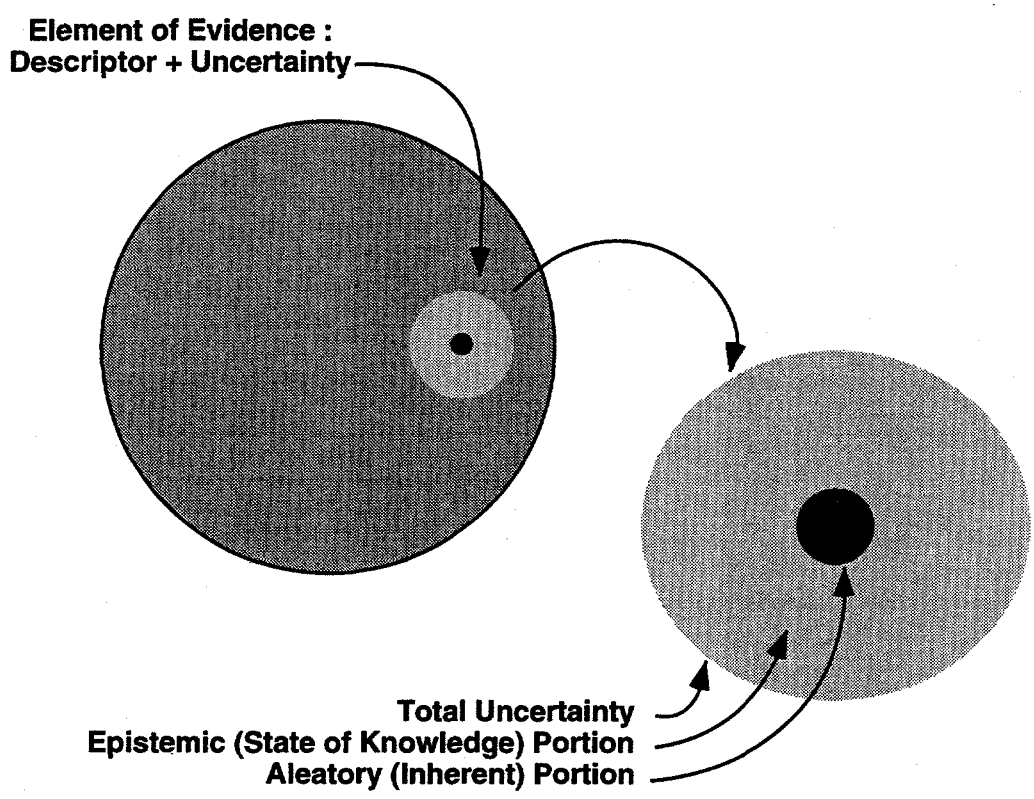

Fig. 2-5. Relationship between components of uncertainty for an element of evidence.

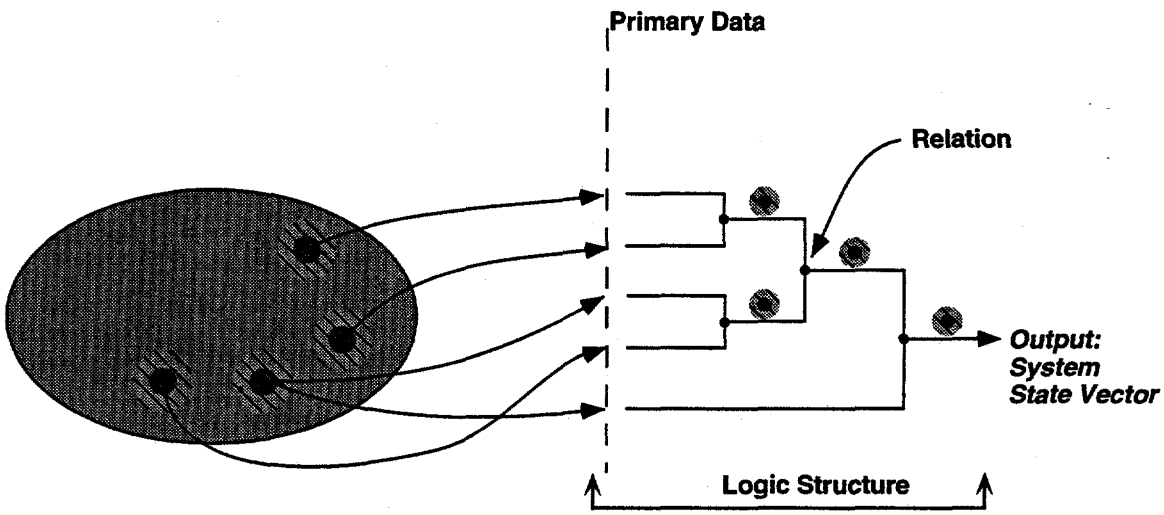

Fig. 2-6. Relationship between a body of evidence and the associated inductive logic structure.

The pressure correlation and the level change in this example are linguistic variables and are the antecedents of the implication. The conclusion variable is "the presence of a significant quantity of retained gas." We will discuss why this particular consequent is used in Sec. 3. As noted earlier, linguistic variables are allowed to have membership in as many fuzzy sets as are needed so that a reasonably complete description of the quantity is possible. For example, the pressure correlation C_p, might have membership in the fuzzy sets {Poor}, {Fair}, and {Good}, and the level change Δh, defined on the universe of discourse {{Small}, {Medium}, {Large}}, written as

$$C_p \in \{\{\text{Poor}\}, \{\text{Fair}\}\{\text{Good}\}\}$$

$$\Delta h \in \{\{\text{Small}\}, \{\text{Medium}\}\{\text{Large}\}\}$$

It can be seen that in this case, we need 3 x 3 = 9 different implications to cover all the possible combinations of the two antecedents. We refer to this set of implications as a "rule base." The complete form of the inference rule is

"(A is A_i and B is B_j) and (If A_i and B_j imply C_k) then C_k"

or

$$[(A_i \wedge B_j) \wedge ((A_i \wedge B_j) \rightarrow C_k)], C_k \; .$$

This statement is a special logical construct known as the *modus ponens* tautology and is the basic form of rule base used in all AR models. Implication 1 above is of exactly this form.

Statements such as those given above are evaluated algorithmically in an AR model. The effects of ambiguity and imprecision are incorporated by the use of fuzzy sets, and the expert judgment required is represented in a series of forward-chaining *modus ponens* rule bases. This allows a computer-based implementation.

2.4. Illustration of Approximate-Reasoning Based Flammable Gas Watch List Screening

To illustrate the operation of an AR model, we will use a short excerpt from the complete FGWL screening algorithm to be described in Sec. 3. The absolute level of the waste in a tank can provide information on the amount of retained gas under the correct circumstances. A substantial difference between the measured waste level and the waste level predicted by the fill/transfer history of the tank corrected for evaporation can be evidence of gas retention in the waste. The greater the unexplained level change, Δh, the greater the potential volume of trapped gas. This model is conceptually simple, but its application can be difficult. All waste transfers and other losses from the tank, including evaporation, must be accounted for. Given the state of the historical records, the large uncertainty in level measurements for some sensors, and the possibility of slow leaks or intrusions, this can rapidly become complex, and in some cases, the results to be inferred from it are problematic.

Hopkins (1994) defines the effective long-term level change to be attributed to retained gas as*

$$\Delta h = h' - h_{81} + \Delta h_{81} + \Delta h_E = \Delta h_M + \Delta h_{81} + \Delta h_E \ , \tag{2-1}$$

where

h' = the recently measured level corrected for transfers since 1981,
h_{81} = the level measured in 1981 (used as a datum),
Δh_{81} = the estimated gas retention level change before the 1981 measurement, and
Δh_E = a correction to the level to account for evaporation after 1981.

The difference between the first two terms in Eq. (2-1) is the measured level change denoted by Δh_M. The value of Δh as a predictor of retained gas depends to a large degree on how large the correction terms Δh_E and Δh_{81} are relative to Δh_M. If these correction terms are large, it is reasonable to discount the importance of this model prediction. This is exactly the type of expert judgment that an AR model is designed to emulate.

We wish to draw an inference in this example about the likelihood of a significant quantity of retained gas. To determine this likelihood, $L_{\Delta h}$, both the unexplained level change, Δh, and the quality of the data used to calculate this parameter should be evaluated. The logic structure for this evaluation is shown in Fig. 2-7. The three inputs are the long-term level change and two parameters, M_{81} and M_E, used to measure the effect of correction terms on the estimate for Δh. These two parameters act as antecedents to allow us to infer a single quality parameter, Q. Q and the level change, Δh, are then in turn the antecedents used to infer a consequent likelihood of a significant quantity of retained gas, $L_{\Delta h}$. This chaining of inferences is characteristic of AR models.

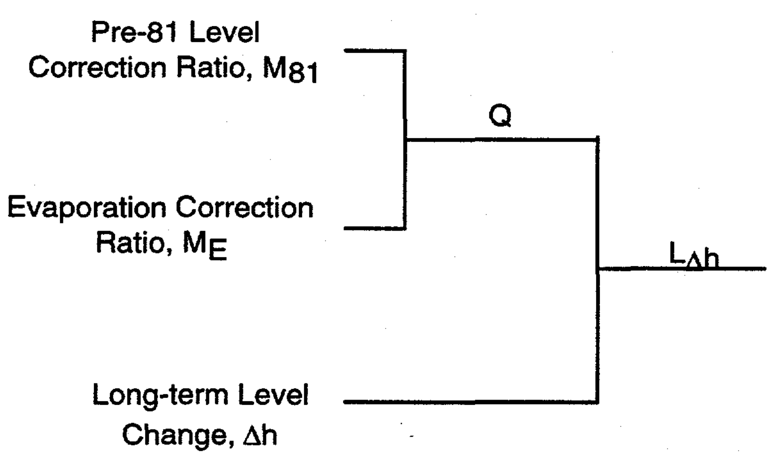

Fig. 2-7. Logic structure for determination of long-term level change predictor likelihood, $L_{\Delta h}$.

*Other factors that may affect Δh have been neglected here.

The relative importance of Δh_E. and Δh_{81} in determining Δh is represented by the parameters M_{81} and M_E, which are defined as

$$M_{81} = |\Delta h_{81} / \Delta h_M| \tag{2-2}$$

and

$$M_E = |\Delta h_E / \Delta h_M| \tag{2-3}$$

The larger the absolute value of these ratios, the larger is the influence of the poorly known parameters, Δh_E and Δh_{81}.

To proceed further, it is necessary to first define the fuzzy sets to which the two correction terms may belong. In this case, we assert that both terms may belong to (that is, have membership in) the same three sets: $\{M_E, M_{81}\} \in \{\{\text{Small}\}, \{\text{Medium}\} \{\text{Large}\}\}$. In practice, the number of sets and the actual set linguistics are developed in conjunction with subject matter experts. The two correction terms initially will be expressed numerically, so we need to develop membership functions to assign the DOM in each set for a particular value of the correction term. One possible set of membership functions is shown in Fig. 2-8.

If $M_E = 3$ (that is, the evaporation correction is three times larger than the measured level change), then M_E is said to have DOMs, γ_E, in the three fuzzy sets of $\gamma_E = \{0,.5,.5\}$. That is, the DOM in {Medium} is $\gamma(M_E, \text{Medium}) = 0.5$. Similarly, a value of $M_E = 0.25$ would imply DOMs of $\gamma_E = \{.75,.25,0\}$. In natural language, this might be expressed as "the evaporation correction ratio is fairly small."

Given the antecedents M_E and M_{81} and the fuzzy sets to which they belong, we are prepared to define a set of expert judgments that relate them to the quality, Q, of the long-term level change prediction. We chose to use the fuzzy sets {{Poor}, {Fair} and {Good}} to describe Q, $Q \in$ {{Poor}, {Fair} and {Good}}. It is not necessary to define membership functions for Q because it is not itself an element of evidence and exists only as an internal linguistic variable. There are nine rules in the *modus ponens* rule base for Q; these are shown in Table 2-4. The shaded box corresponds to the rule:

> "IF the evaporation correction ratio is *medium* AND the pre-1981 level change correction ratio is *medium* THEN the quality of the unexplained level change model is *fair*".

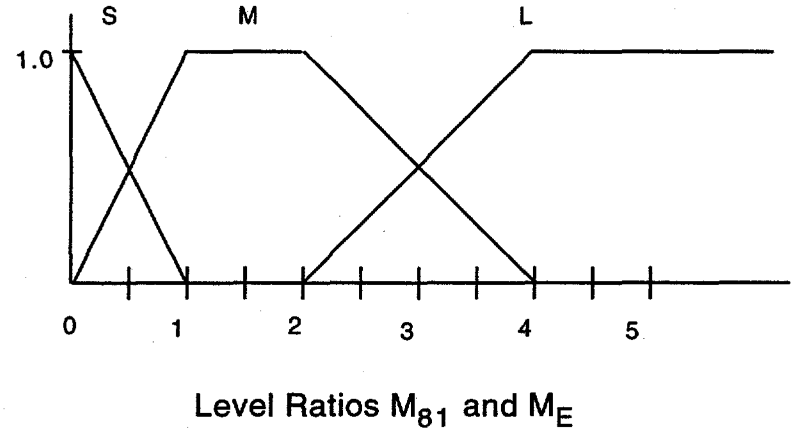

Fig. 2-8. M_{81} and M_E ratio membership functions.

Table 2-4
***Modus Ponens* Rule Base for M_{81} and M_E to Determine Q**

Q Rules

		M_E: S	M_E: M	M_E: L
M_{81}	L	F	P	P
	M	F	F	P
	S	G	F	P

Referring to Fig. 2-7, the next inference is made about $L_{\Delta h}$ using Q and the value of Δh itself. Again we define the fuzzy sets in which Δh may have membership: $\Delta h \in$ {{Very Small},{Quite Small}, {Moderate}, {Quite Large}, {Very Large}} and the associated membership functions shown in Fig. 2-9. The likelihood of retained gas, $L_{\Delta h}$, is a linguistic variable that we choose to characterize by its membership in a series of sets that describe degree of likelihood:

$L_{\Delta h} \in$ {{Very Unlikely},{Quite Unlikely},{Unresolved},{Quite Likely},{Very Likely}}.

The use of the hedges "quite" and "very" is consistent with the expressions commonly used by subject matter experts. It must be emphasized that "Unresolved" does not mean "Equally Likely" but rather "No definite statement can (or should) be made." The rule base for inferring $L_{\Delta h}$ is given in Table 2-5. In particular, note the bottom row in the rule base. If the Quality is poor, then $L_{\Delta h}$ always evaluates to "Unresolved." This row of the rule base deals with the situation in which the quality of the data does not allow a strong conclusion to be reached with this model.

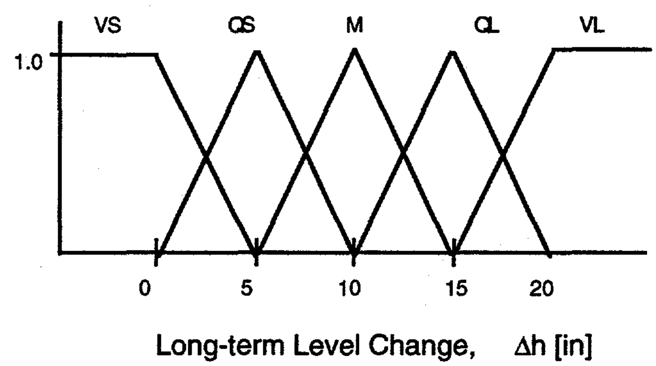

Fig. 2-9. Long-term level change Δh membership functions.

Table 2-5
***Modus Ponens* Rule Base for Q and Δ_h to Determine $L_{\Delta h}$**

		Δh: VS	Δh: QS	Δh: M	Δh: QL	Δh: VL
Q	G	VU	QU	U	QL	VL
	F	QU	U	U	U	QL
	P	U	U	U	U	U

Consider now a numerical example using this set of linguistic variables, membership functions, and rule bases. We will assume that the following differential level data are available.

$\Delta h_M = 2.8$ in.

$\Delta h_{81} = 1.4$ in.

$\Delta H_E = 8.3$ in.

These correspond to $M_{81} = 0.5$, $M_E = 3$ and $\Delta h = 12.5$ in. The DOMs for M_{81} and M_E from Fig. 2-8 are $\gamma_{81} = \{.5,.5,.0\}$ and $\gamma_E = \{0,.5,.5\}$, respectively. This means that four of the rules in Table 2-4 will be operative—the lower two rows by the two rightmost columns. We say that these four rules "fire." The firing of *modus ponens* rule bases with fuzzy antecedents is determined using the "min-max" rule. The details of the operation of this rule are discussed in Appendix A, which gives the details of the various operations associated with AR theory. For Q, application of the rule yields $\mu_Q = \{.5,.5,0\}$; that is, the quality has equal membership in the {Poor} and {Fair} sets. We could express this as "the quality is poor to fair," which reflects the judgment incorporated into the rule base that if either ratio is large, then the quality cannot be good. The DOMs for $\Delta h = 12.5$ in. are $\gamma_{\Delta h} = \{0,0,.5,.5,0\}$. That is, the level change has non-zero membership only in {Moderate} and {Quite Large}. Evaluation of the rule base for $L_{\Delta h}$ using Q and Δh as antecedents with the min-max operator yields $\gamma_{L\Delta h} = \{0,0,.5,0,0\}$. With this set of data, there is only non-zero membership for $L_{\Delta h}$ in the likelihood fuzzy set {Unresolved}, so the conclusion is that the likelihood of a significant quantity of retained gas using the long-term level change is "unresolved." This agrees with the premise stated earlier that if correction terms are large, then the inference based on the long-term level change must be weak. Figure 2-10 shows how DOMs are propagated as the rule bases act on the elements of evidence.

The fuzzy set membership vector $\gamma_{L\Delta h} = \{0,0,.5,0,0\}$ describes the gas retention likelihood state of the tank using an evaluation based on the long-term level change. We recognize this as a state vector, and in this case, its interpretation is straightforward. However, consider the vector $\gamma_{L\Delta h} = \{0,0,.5,.8,.75\}$. What does this mean in terms of the likelihood of retained gas and how can this be compared with a screening

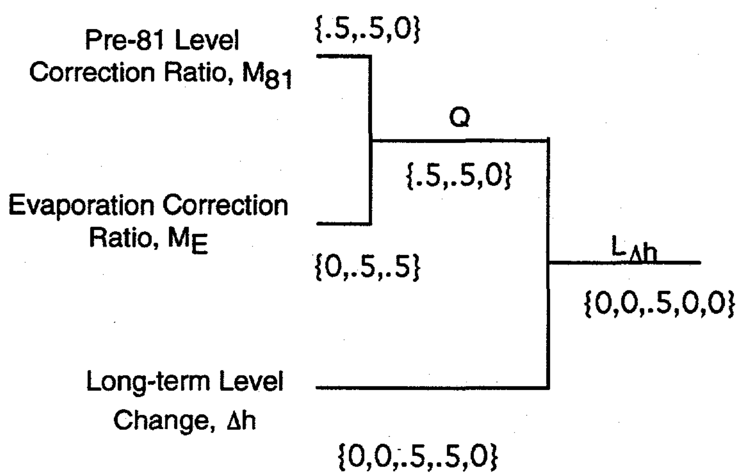

Fig. 2-10. Propagation of DOMs for example evaluation.

criterion? The answer to both of these questions requires that we convert the likelihood fuzzy set membership vector to a single measure. This is referred to as "defuzzification." In Sec. 3, we will discuss several techniques for defuzzification. One approach is to take the maximum DOM as the output:

$$D(\gamma L\Delta h) = \max[\gamma L\Delta h] \ .$$

For the second example, this is $D_{(\gamma L\Delta h)} = 0.8$. Further, this value is associated with the set {Quite Likely}. Therefore, the output of the logic module is "quite likely." We can now define a screening criterion that uses the same natural language expressions.

> If the evaluation output is contained within {Very Unlikely, Quite Unlikely}, the tank passes the screening.
>
> If the evaluation output is contained within {Very Likely, Quite Likely}, the tank fails the screening.
>
> If the evaluation output is contained within {Unresolved}, there is insufficient information to classify the tank.

When these criteria are applied to the two membership vectors above, the screening results are $\gamma_{L\Delta h}$ = {0,0,.5,0,0} $\rightarrow$ "insufficient information to classify the tank" and $\gamma_{L\Delta h}$ = {0,0,.5,.8,.75} $\rightarrow$ "the tank fails the screen." Of course, the three primary inputs to this short AR model, Δh_M, Δh_E, and Δh_{81}, all have uncertainty associated with them. In the case of Δh_M, the dispersion may be assigned primarily to various forms of measurement error. The uncertainty in Δh_E and Δh_{81} is probably mainly epistemic. As a consequence, the likelihood vector and the screening result are also random variables. We will discuss how this uncertainty is taken into account at the conclusion of Sec. 3, where the detailed AR model for flammable gas screening is developed using the techniques illustrated in this example.

3.0. FLAMMABLE GAS WATCH LIST SCREENING MODEL DEVELOPMENT

In this section, we develop a complete AR model for FGWL screening. This model is intended to illustrate the concepts used in the AR method and to provide a credible basis for testing the basic algorithm. The following major points will be covered.

- Specification of the scope of the model and the form in which the result of the evaluation is to be expressed.
- Presentation of the inductive logic structure for screening. This defines the relationship between the data, models, and judgments used and the order in which the inferences are to be made.
- Definition of linguistic variables for the elements of evidence. Here we define the fuzzy sets used to describe each element and present illustrative membership functions needed to transform input parameters into the appropriate set membership.
- Development of the implication rule bases that prescribe the inferences to be drawn at each branch point in the logic tree.
- Explanation of the methods used to express the uncertainty associated with the evaluation.

Each of these points addresses an essential aspect of the AR methodology and its application to the FGWL screening process. The discussion of the linguistic variables and the implication rule bases are ordered according to the primary evaluation modules in the logic structure.

This section describes the details of the pilot AR model in considerable depth. Readers uninterested in this amount of detail can gain a reasonable understanding of how the AR approach has been adapted to the FGWL screening problem by reading Secs. 3.1 and 3.2, the introduction to Secs. 3.3 and 3.3.1, the introductions to Secs. 3.4 and 3.5, and all of Secs. 3.5 through 3.8.

3.1. Specification of the Screening Evaluation Output

An important consideration in the initial development of an AR model is its scope. The scope determines the size of the required logic structure and is a major determinant in the amount of work associated with developing rule bases and membership functions. To illustrate the AR approach to FGWL screening, we chose to restrict ourselves to an evaluation of retained gas. We had two reasons for this decision. First, the body of evidence concerning gas generation and retention appears to be generally more mature than that associated with other aspects of flammable gas phenomenology. Second, the body of evidence for retention provides a diverse set of data and models that is sufficient to illustrate the ability of an AR model to combine quantitative and qualitative information and make sophisticated judgments about model validity and the resolution of conflicting results.

A second important consideration in designing an AR model is determining the form in which the final output of the AR model is to be expressed. This is equivalent to specifying the format in which a subject matter expert is expected to state his conclusions. Ideally, the natural language expressions associated with the output of the model are developed in conjunction with the experts used in building it. The linguistic variable chosen for the final output in the pilot model was "likelihood of a significant quantity of retained gas." The adjective "significant" means that there is sufficient gas retention so that concerns in terms of the Wyden amendment exist. We express the output likelihood, hereafter referred to as the aggregate likelihood or L_F, with the following set of descriptors: "Extremely Unlikely,, "Very Unlikely," "Quite Unlikely," "Unresolved," "Quite Likely," "Very Likely," and Extremely Likely." We will show how these descriptors are used directly in AR. The hedges "extremely," "very," and "quite" are intended to provide sufficient resolution to allow meaningful distinctions to be made. The set {Unresolved} is used to allow for expression of evaluations where the results are inconclusive. This is equivalent to an expert saying "I don't know" or "The data are inconclusive." We use the expression "likelihood" in the sense that it "supplies a natural order of preference among the possibilities under consideration" (Thomas 1995). That is, something that is said to be "very likely" is understood to have a more realistic chance of happening or to occur more frequently than something that is "extremely unlikely." However, it must be emphasized that the likelihood linguistic variable is not to be confused with quantitative probability nor do we intend our use of likelihood to be associated directly with the likelihood function of probability theory.

3.2. Overview of the Inductive Logic Structure

The parameters used in the algorithm are grouped into three general classes: predictors, enablers, and indicators. These three classes correspond to the primary modules in the inductive logic tree as shown in Fig. 3-1. Each class of parameters provides a distinct judgment concerning the likelihood of a significant quantity of retained gas. We refer to these likelihoods as L_P, L_E, and L_I, where the subscripts denote the predictor, enabler, and indicator parameter groups. Each of these three linguistic variables represents an independent evaluation of the likelihood for a significant quantity of retained gas based on a particular combination of logical inferences. Predictor and enabler likelihoods and their subsidiary likelihoods are defined in terms of the fuzzy sets described in Sec. 2.4:

{{Very Unlikely}, {Quite Unlikely},{Unresolved}, {Quite Likely} ,{Very Likely}} .

We refer to this set of sets as the universe of discourse and require that each of these linguistic variables have a non-zero DOM in at least one of the sets in the universe of discourse. The set theoretic shorthand for this is

$L_P \in$ {{Very Unlikely},{Quite Unlikely},{Unresolved},{Quite Likely}, {Very Likely}} .

The corresponding sets for L_I are

$L_I \in$ {{Extremely Unlikely},{Unresolved},{Extremely Likely}} .

The use of sets with the hedge "extremely" for L_I is intended to reflect the fact that indicators can be especially clear elements of evidence, and therefore, the associated inferences may be particularly strong. The deletion of the "quite" and "very" hedges incorporates the companion decision to *only* value indicator evidence if it is particularly unambiguous. This is done for illustrative purposes, and for the tests discussed in Sec. 5, all inputs to the indicator module were chosen to yield only membership in {Unresolved} for L_I .

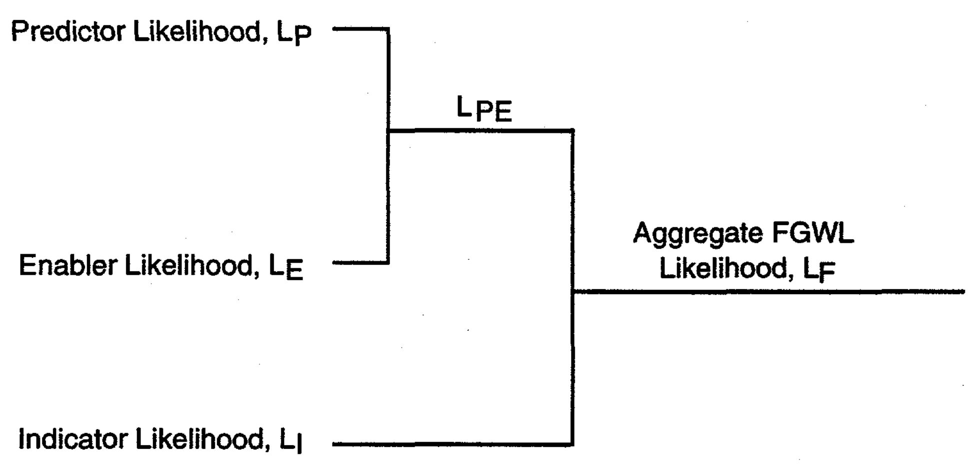

Fig. 3-1. Logic for combining likelihood judgments based on predictor, enabler, and indicator parameter classes.

The three major likelihood judgments are combined according to the logic structure in Fig. 3-1. The inputs to this structure are actually the final outputs of more involved evaluations to be discussed in detail shortly. The output of the final rule, L_F, represents the expert judgment for a tank based on all three classes of parameters. The fuzzy sets to which L_F can belong are the union of the universes of discourse for L_P, L_E and L_I:

$L_F \in$ {{Extremely Unlikely},{Very Unlikely},{Quite Unlikely}, {Unresolved},{Quite Likely}, {Very Likely}, {Extremely Likely}}.

The fuzzy sets for each of these likelihoods, their abbreviations, and the rule bases in which they appear are given in Table 3-1 for reference.

Predictor parameters include the barometric pressure-level correlation and the long-term level change; the retained gas sampler and voidmeter sensors are not considered in the current illustrative AR logic structure. Both of the parameters considered here use a measurement to provide an estimate of the amount of gas trapped in the waste. Enablers are sets of parameters that, when properly combined, provide a basis for estimating the gas generation rate and the gas retention effectiveness for a tank. Gas indicators are parameters that can be used to infer the existence of a GRE. Positive indicators are direct measurements of an unambiguous nature, such as a dome-space flammable gas concentration measurement. The absence of such positive indicators does not prove that a tank is a nonflammable gas tank. Similarly, a negative indicator has a threshold value that indicates conclusively that significant gas retention is not possible in the tank because of some distinct combination of physical characteristics of the waste and tank. We use the WHC "Quick Screen" criterion as an example of a negative indicator.

Table 3-1
Summary of Linguistic Variables for Primary Logic Modules

Parameter	Symbol	Universe of Discourse	Set Abbreviations	Rule Base Tables
Predictor Likelihood	L_P	{{Very Unlikely}, {Quite Unlikely}, {Unresolved}, {Quite Likely}, {Very Likely}	{VU, QU, U, QL, VL}	3-12 3-35
Enabler Likelihood	L_E	{{Very Unlikely}, {Quite Unlikely}, {Unresolved}, {Quite Likely}, {Very Likely}}	{VU, QU, U, QL, VL}	3-23 3-35
Indicator Likelihood	L_I	{{Extremely Unlikely}, {Unresolved}, {Extremely Likely}}	{EU, U, EL}	3-34 3-36
Aggregate FGWL Likelihood	L_F	{{Extremely Unlikely}, Very Unlikely}, {Quite Unlikely}, {Unresolved}, {Quite Likely}, {Very Likely}, {Extremely Likely}}	{EU, VU, QU, U, QL, VL, EL}	3-36

Figure 3-2 shows the complete inductive logic structure for the FGWL screening model. The major submodules correspond to the three primary likelihoods, L_P, L_E, and L_I.. A detailed discussion of the inference module for each of these likelihoods is given below. We will return to the process used in their aggregation to infer L_F in Sec. 3.6.

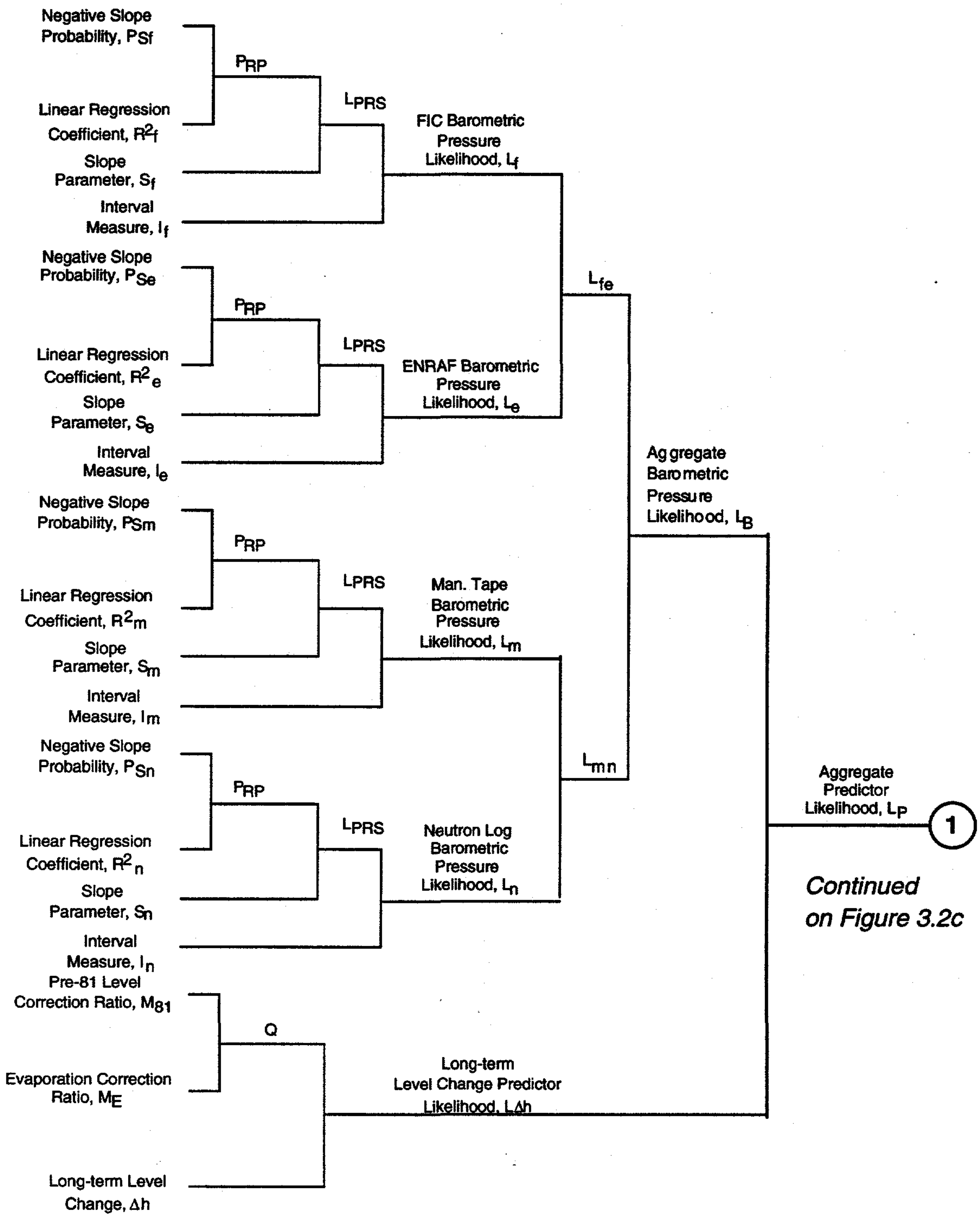

Fig. 3-2(a). Complete logic structure for evaluating the likelihood of a significant quantity of retained gas.

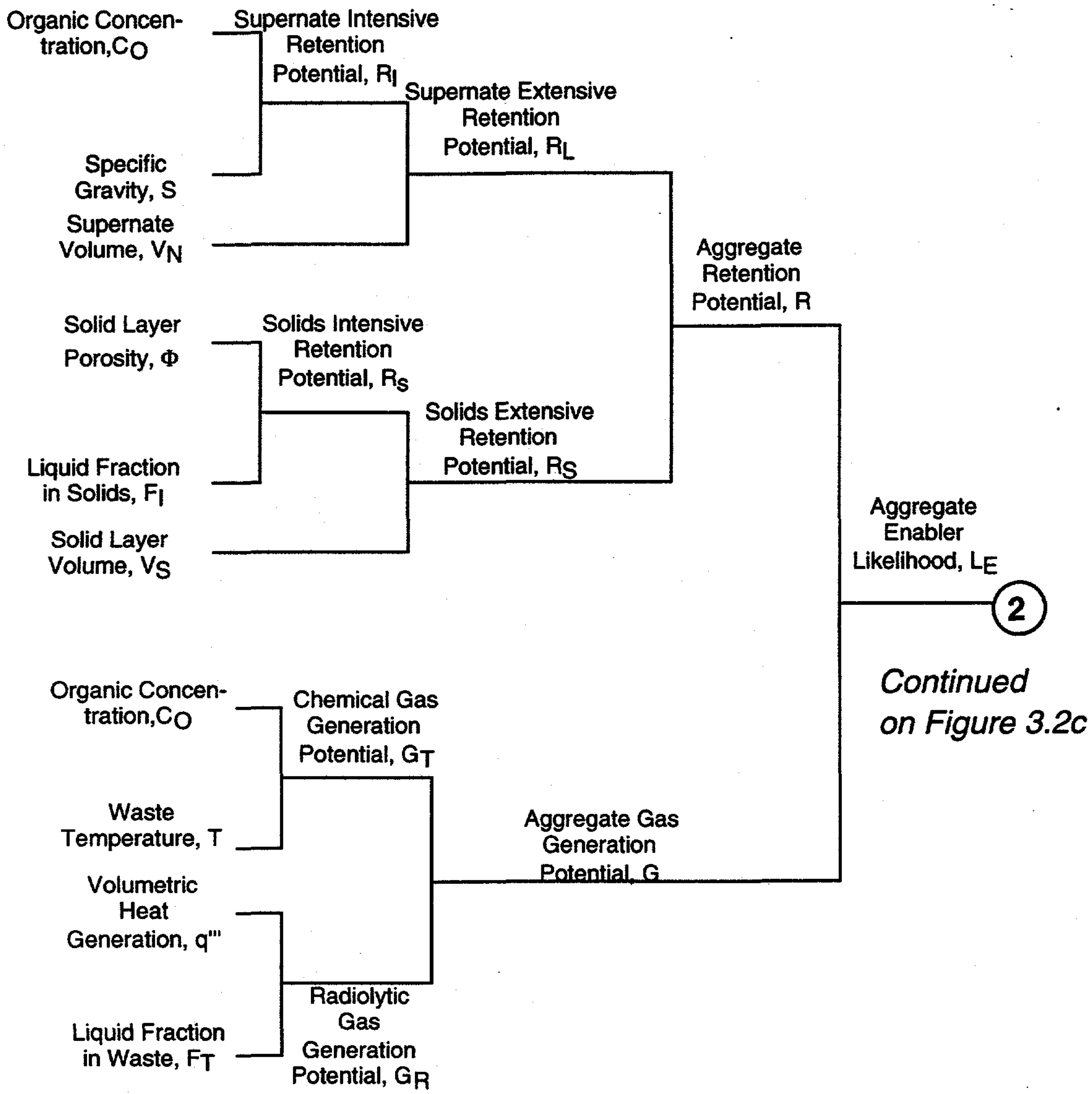

Fig. 3-2(b). Continued.

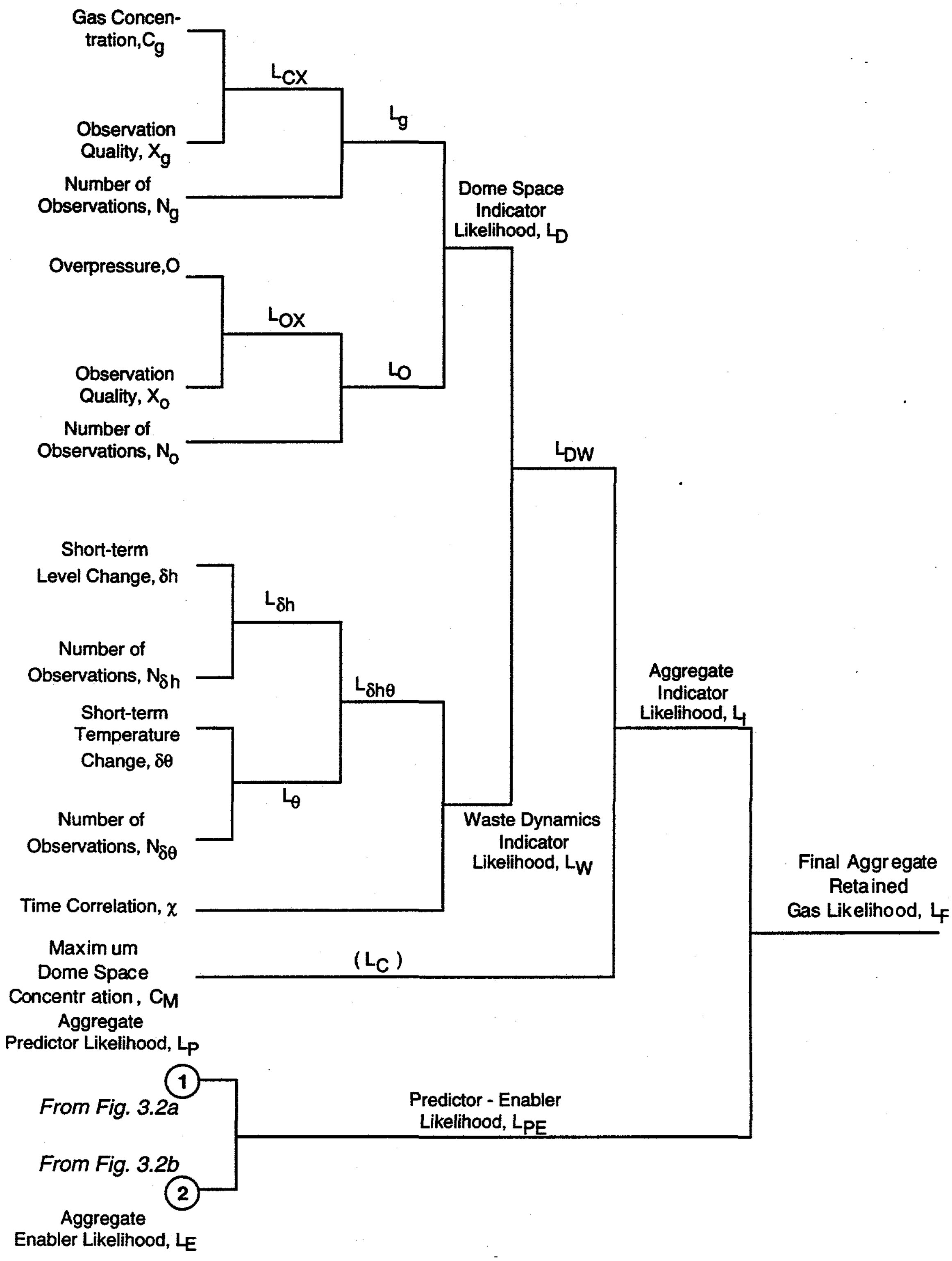

Figure 3-2(c). Continued.

3.3. Predictor Likelihood Logic Module

Figure 3-3 shows the complete logic structure used to calculate the predictor likelihood. The two basic models used are the correlation between barometric pressure and waste level fluctuations (Whitney 1995) and the long-term level change discussed earlier. Other sensors that provide a prediction of retained gas volume also could be included.

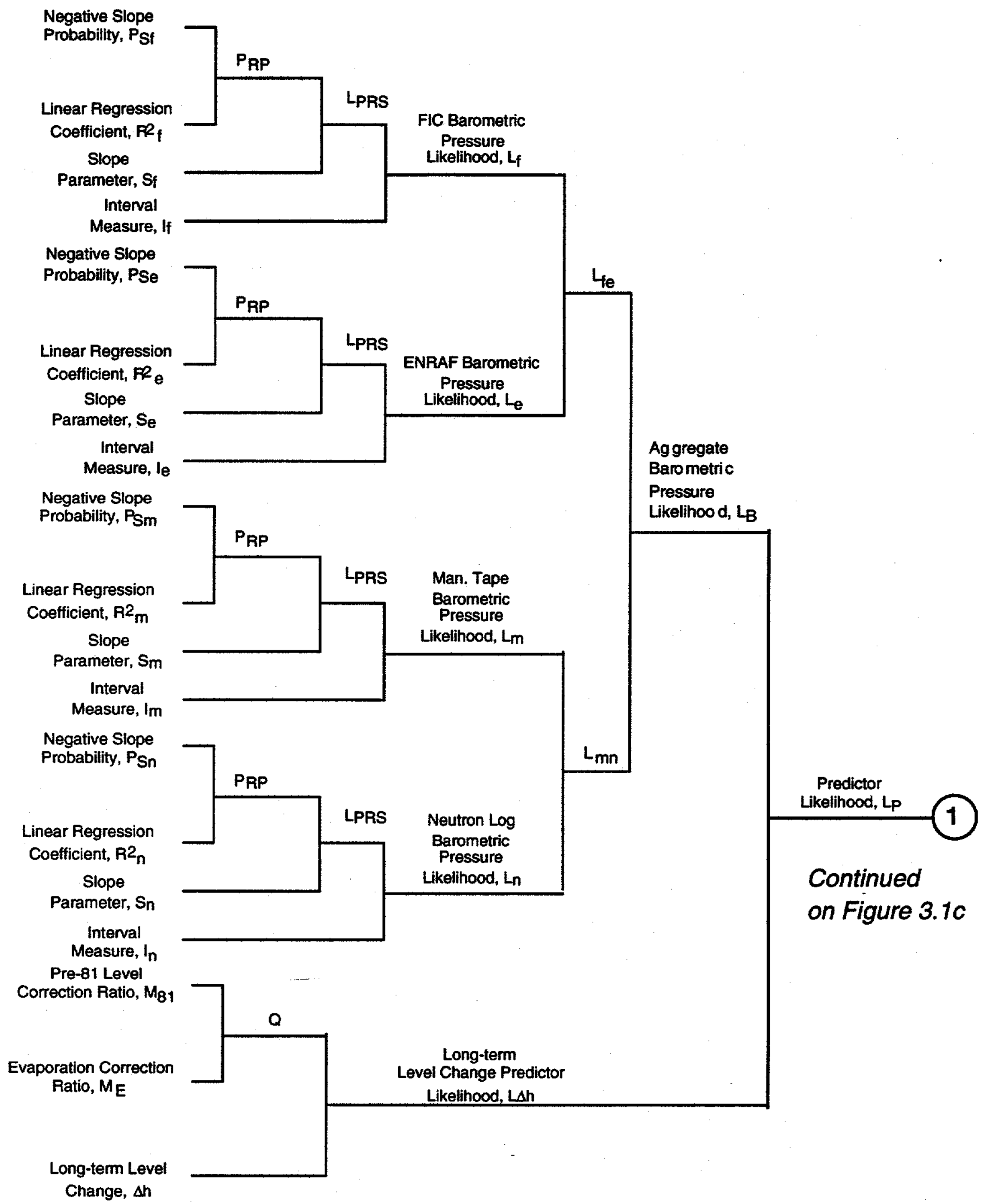

Fig. 3-3. Predictor likelihood logic module.

3.3.1. Surface Level—Barometric Pressure Logic Submodule. The barometric pressure likelihood, L_B, is determined from an evaluation of the available evidence on the correlation between fluctuations in barometric pressure and the waste level. The level fluctuation is assumed to be related to the pressure fluctuation by

$$dL = S\, dP \ , \qquad (3\text{-}1)$$

where dL is the level change, dP is the change in pressure, and S is the slope of a linear least-squares fit of the level and pressure data. Because an increase in pressure leads to a reduction in gas volume (at constant temperature) and hence a drop in level, the slope, S, is expected to be negative when gas is retained. The value for S is obtained by a linear least-squares fit of the (L,P) data. The basic assumption in this model is that the more negative the slope, the greater the likelihood of retained gas in the waste. There are four independent ways to calculate the correlation using the available sensors: FIC, ENRAF, manual tape, and neutron log level instruments. All four sensors are not installed in every tank. The logic module for each instrument is the same as shown in Fig. 3-4.* The parameters in Fig. 3-4 are the same for each instrument, and the subscript i can take on the values f, e, m, and n for FIC (f), ENRAF (e), manual tape (m), and neutron log (n), respectively.

Inputs to this module are the following.

Negative Slope Fraction Probability, P_S. If changes in barometric pressure and surface or interstitial liquid level are uncorrelated, then we would expect that the sign of the derivative, dL/dP, would be equally likely to be positive or negative. P_S is the probability of getting the observed fraction of negative correlation events assuming that no correlation exists. If the L and P values are uncorrelated, the probability that S is less than zero is 0.5 for each interval. Then, using the binomial distribution, we may calculate the probability that there are N negatively correlated intervals in I total intervals as

$$P_S = I! \,/\, N! \,/\, (I - N)! * 0.5^I \qquad (3\text{-}2)$$

A value of $P_S = 0.05$ is used by Whitney as a flag for retained gas. Note that if I is very small, P_S will be relatively large even if N is close to I. In this case, it is not possible to draw a strong inference from P_S. For large I, N must be a significantly larger fraction of I than I/2 for P_S to be small. We define P_S on the universe of discourse, {{Low}, {Medium}, {High}}, ($P_S \in$ {L,M,H}).

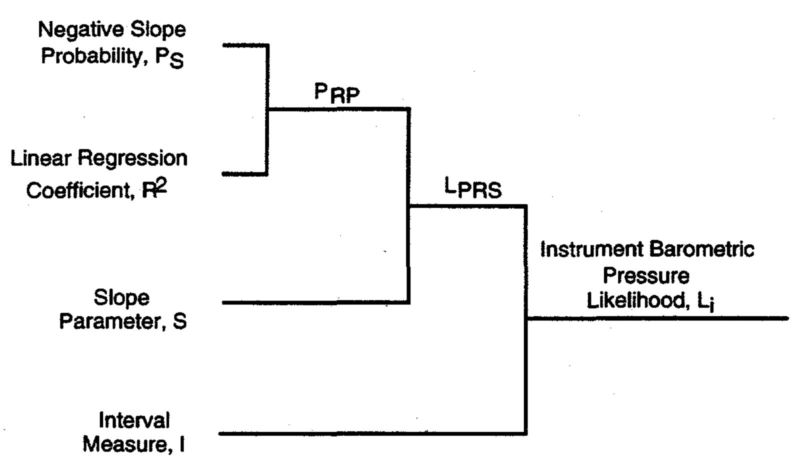

Fig. 3-4. Barometric pressure logic submodule for individual sensor.

*An alternative logic structure would relate S and R^2 first.

Linear regression coefficient, R^2. If the level and pressure fluctuations vary exactly in a simple linear relationship, then the regression coefficient will be 1.0. Similarly, if there is no correlation, $R^2 = 0$. Thus, the closer R^2 is to 1.0, the more evidence there is for the assumed linear behavior consistent with gas retention. The universe of discourse for the regression coefficient is {{Poor}, {Fair}, Good} ($R^2 \in \{P,F,G\}$).

Slope coefficient, S . This is the value obtained from the best fit to the relationship dL = S dp. By making a number of assumptions, the value of S also can be used to estimate a volume of gas that, if it were to act as a simple compressible region, would produce the observed effect. This is the basis for the previously mentioned predictor capability associated with an inference based on the barometric pressure correlation. A large negative slope in this model implies a large amount of retained gas.* The universe of discourse for S is {{Positive}, {Slightly Negative}, {Very Negative}}, (($S \in \{P,SN,VN\}$).

Number of intervals, I. This is the number of intervals used to calculate P_S. The value of I depends on the historical data base as well as the results of an algorithm used to remove periods where the signals' behavior is unsuitable for regression analysis. I is defined on the universe of discourse {{Low}, {Medium}, High}, ($I \in \{L,M,H\}$).

The universe of discourse used to express each of these parameters as a linguistic variable is summarized in Table 3-2 along with references to the inference rule bases in which each input element appears. Membership functions for each of these parameters are shown in Fig. 3-5. The regression coefficient and slope parameter are random variables; the other two quantities are constants. Note that a different membership function is used for the ENRAF interval count. This allows for the fact that this instrument has been fielded for only a relatively short period of time. However, the correlation calculated with this level sensor is considered of high quality even though the actual number of intervals is usually small relative to the other sensors.

Implication Rule Bases for L_B. An inference rule base is constructed for each implication junction in Fig. 3-4. The basic principle here is that the likelihood of gas is greater when P_S, R^2, and S are all in agreement and that the weight given to this judgment depends on the number of intervals on which the statistics are based. The first step in the level-pressure screening parameter process

Table 3-2
Summary of Input Elements for Individual Sensor Barometric Pressure Logic

Parameter	Symbol	Universe of Discourse	Set Abbreviations	Rule Base Table	Membership Function Figure
Negative slope fraction probability	P_S	{{Low},{Medium},{High}}	{L,M,H}	3-3	3.5
Regression coefficient	R^2	{{Poor},{Fair},{Good}}	{P,F,G}	3-3	3.5
Slope	S	{{Positive},{Slightly Negative},{Very Negative}}	{P,SN,VN}	3-4	3.5
FIC, manual tape and neutron log interval count	I	{{Low},{Medium},{High}}	{L,M,H}	3-5	3.5
ENRAF interval count	I	{{Low},{Medium},{High}}	{L,M,H}	3-5	3.5

*In his report, Whitney did not recommend computing gas volumes from the slope values.

Fig. 3-5 (Quark Figure)

infers a gas probability parameter denoted by P_{RP} from antecedents P_S and R^2. The rule base for this implication set is shown in Table 3-3. This is classified as a conflation rule base because we are drawing an inference from two dissimilar quantities. There are several different types of conflation rules. The various classes of implication rule bases are discussed in Appendix B. The universe of discourse for P_{RP} is the same as for P_S, $P_{RP} \in$ {{Low}, {Medium}, {High}}. However, for P_{RP}, membership in {High} is a positive indication of retained gas. In this rule, we follow the expert consensus that P_S is a strong indicator of retained gas. A difference in the analysis here is that the regression coefficient R^2 is also considered to be an important measure of the probability for retained gas. When the linear correlation is considered good, then low values of P_S are confirmed and the judgment for P_{RP} is membership in {High}. Similarly, the combination [$P_s \in$ {High}, $R^2 \in$ {Poor}] confirms a lack of retained gas and leads to the judgment $P_{RP} \in$ {Low}. A fair correlation is indicative of ambiguity and moves the gas likelihood toward the medium set. Note that no membership functions have been defined for P_{RP}. It exists as a pure linguistic variable. Membership functions are only required for the numerical elements of evidence used as inputs to the logic structure.

Table 3-3
Conflation Rule Base for P_{Si} and R^2_i to Generate P_{RPi}

P_{RPi} Rules

		P	F	G
P_{Si}	H	L	M	M
	M	L	M	H
	L	M	H	H

R^2_i (columns)

After P_{RP} is inferred, it is combined with the slope parameter S calculated by Whitney (1995) to generate an instrument-dependent gas likelihood, L_{PRS}. The rule base for combining P_{RP} and S to infer L_{PRS} is shown in Table 3-4. The rule base in our model is symmetrical, meaning that equal weight is given to both antecedents. The most important property of this rule is transformation. That is, it transforms the inputs into an output that is different from either. Here the likelihood sets associated with the consequent use linguistic descriptors associated with likelihood. This transformation occurs in each major branch of the logic structure and represents the transition from diverse input data types to likelihood measures. The universe of discourse for L_{PRS} is

{{Very Unlikely},{Quite Unlikely}, {Unresolved},{Quite Likely}, {Very Likely}}

and is used to represent all of the likelihood linguistic variables used in the predictor logic module.

Table 3-4
Conflation Rule Base for P_{RPi} And S_i to generate L_{PRSi}

L_{PRSi} Rules

		P	SN	VN
P_{RPi}	H	U	QL	VL
	M	QU	U	QL
	L	VU	QU	U

S_i (columns)

A gas likelihood for each instrument, L_i, is constructed by modifying L_{PRSi}, by the number of intervals, I_i. Recall that I is defined on the universe $I \in$ {Low, Medium, High}. The implication rule base for L_{PRS} and I is shown in Table 3-5. The output from the rule is the individual instrument gas likelihood, symbolized by L_i, and represents the likelihood of significant retained gas in the tank based on the level/pressure data for instrument i. This rule is structured so that if the number of intervals has membership in either {Medium} or {Low}, the judgment for L_i is relaxed toward {Unresolved}. This is consistent with a best-estimate judgment. Note that the judgment about the importance of the number of intervals associated with the ENRAF was incorporated into its membership function rather than by developing a separate rule base. This illustrates the way that decisions about how to express an input linguistically can be separated from the development of implications associated with that variable.

Table 3-5
Conflation Rule Base for I_i and L_{PRSi} to Generate L_i

I_i		L_i Rules				
	H	VU	QU	U	QL	VL
	M	QU	U	U	U	QL
	L	U	U	U	U	U
	L_{PRSi}	VU	QU	U	QL	VL

The next relationship in the inference chain is the combination of the individual sensor judgments, L_i, through a convolution rule to infer L_B, the aggregate barometric pressure likelihood estimate as shown in Fig 3-6. This is referred to as a convolution rule base because it combines two likelihood variables to infer a consequent likelihood. Here the results from the FIC and ENRAF are combined, as are the results of the manual tape (MT) and neutron log (NL). This approach is taken to account for instrument quality. In our pilot model, the FIC and ENRAF are considered to have equal quality in spite of the lower number of intervals for the ENRAF in this context, so their likelihoods are combined symmetrically to produce the likelihood L_{fe} as shown in Table 3-6. An important consideration is that not all sensors are available for each tank. Missing sensors are accounted for by giving them full membership in {Unresolved} when applying the convolution rules.

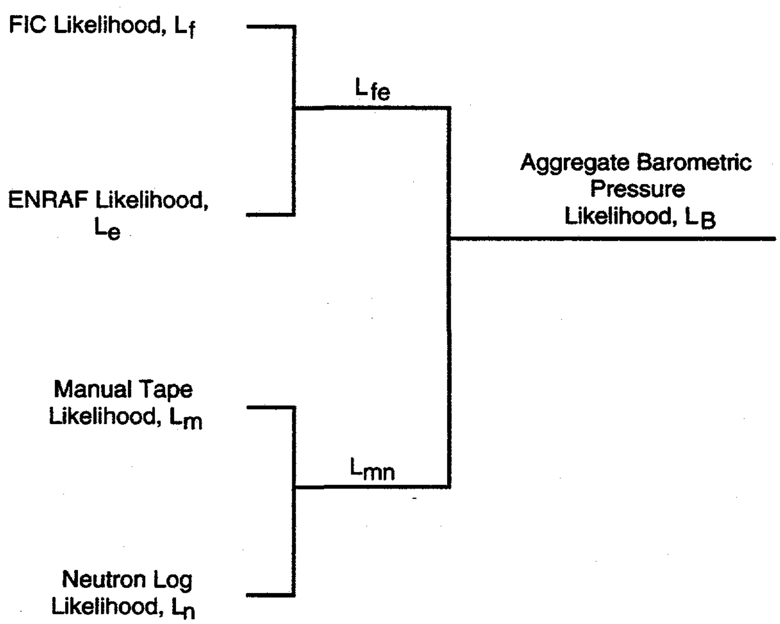

Fig. 3-6. Logic structure for convolution of individual barometric pressure correlation likelihoods.

Table 3-6
Convolution Rule Base for L_f and L_e to generate L_{fe}

L_{fe} Rules

	VL	U	U	VL	VL	VL
	QL	U	U	QL	QL	VL
L_f	U	VU	QU	U	QL	VL
	QU	VU	QU	QU	U	U
	VU	VU	VU	VU	U	U
		VU	QU	U	QL	VL
				L_e		

The MT and NL instruments are judged to have a lower inherent quality. Thus, in the rule base for them (Table 3-7), the universe of discourse for L_{mn} is restricted to {{Quite Unlikely}, {Unresolved}, {Quite Likely}}.

Table 3-7
Convolution Rule Base for L_m and L_n to Generate L_{mn}

L_{mn} Rules

	VL	U	U	QL	QL	QL
	QL	U	U	U	QL	QL
L_m	U	QU	U	U	U	QL
	QU	QU	QU	U	U	U
	VU	QU	QU	QU	U	U
		VU	QU	U	QL	VL
				L_n		

The instrument pair likelihoods, L_{mn} and L_{fe}, are combined to infer a final barometric pressure correlation likelihood, L_B. In this convolution rule base, the relative instrument qualities for the two pairs of instruments are taken into consideration. The rule base for this step is shown in Table 3-8. This represents one possible set of judgments about the relative quality of the different instruments. The *mn* instrument inputs can only modify the *fe* instrument inputs by relatively small amounts in this rule base because the quality of the *mn* pair is judged to be considerably less than that of the *fe* pair. Thus, the gas likelihood is determined principally by the FIC and ENRAF inputs if they are available. If they are not available, then the MT and NL can only make weaker predictions. This is the final inference in the barometric pressure logic submodule, and its output, L_B, is now available for use in determining the overall predictor likelihood L_P.*

Table 3-8
Convolution Rule Base for L_{fe} and L_{mn} to Generate L_B

L_B Rules

	QL	QU	QU	QL	VL	VL
L_{mn}	U	VU	QU	U	QL	VL
	QU	VU	VU	U	QL	QL
		VU	QU	U	QL	VL
				L_{fe}		

*There is an implicit assumption made here that each level sensor signal is correlated only with the pressure change. However, one might expect a temporal correlation between the various level signals in the presence of retained gas. The implication of additional correlation observations could be implemented in a manner similar to that discussed in Sec. 3.4

3.3.2. Long-Term Level Change Logic Submodule. This logic submodule was discussed in Sec. 2. The discussion here provides some additional details concerning $L_{\Delta h}$ and is included for completeness. The absolute level of the waste in a tank can provide information on the amount of retained gas under the correct circumstances and hence is classified as a gas predictor. A substantial difference between the measured waste level and the waste level predicted by the fill/transfer history of the tank corrected for evaporation can be explained by gas retention in the waste. The greater the unexplained level change, Δh, the greater the potential volume of trapped gas. This parameter is conceptually simple, but its calculation is fraught with uncertainty. All waste transfers and water losses from the tank, including evaporation, must be accounted for. Given the state of historical records, the uncertainty in level measurements with some instruments, and the possibility of slow leaks or intrusions, this calculation becomes quite involved, and the relevance of the results can be difficult to assess.

To determine the likelihood of significant gas retention, both the unexplained level change, Δh, and the quality of the data used to calculate this parameter must be evaluated. The logic structure for this evaluation is shown in Fig. 3-7. The three inputs are the long-term level change and two parameters, M_{81} and M_E, used to judge the effect of correction terms on the estimate for Δh. These two parameters are combined to infer a quality parameter, Q. The quality and the long-term level change, Δh, act as antecedents to infer the level change likelihood, $L_{\Delta h}$.

The inputs to this submodule are as follows.

Long-term level change, Δh. The effective Δh is calculated from four tank parameters:

$$\Delta h = h' - h_{81} + \Delta h_{81} + \Delta h_E = \Delta h_M + \Delta h_{81} + \Delta h_E \ , \tag{3-3}$$

where h' is the recently measured level corrected for transfers since 1981, h_{81} is the level measured in 1981 used as a datum point, Δh_{81} is the estimated gas retention level change before the 1981 measurement, and Δh_E is a correction to the level to account for evaporation since the establishment of the measurement datum.* The difference between the first two terms in Eq. (3-3) is denoted by Δh_M. The quality of Δh depends on all three differential levels. The parameter Δh is expressed with the universe of discourse {Very Small}, {Quite Small}, {Moderate}, {Quite Large}, {Very Large}} ($\Delta h \in$ {VS,QS,M,QL,VL}.

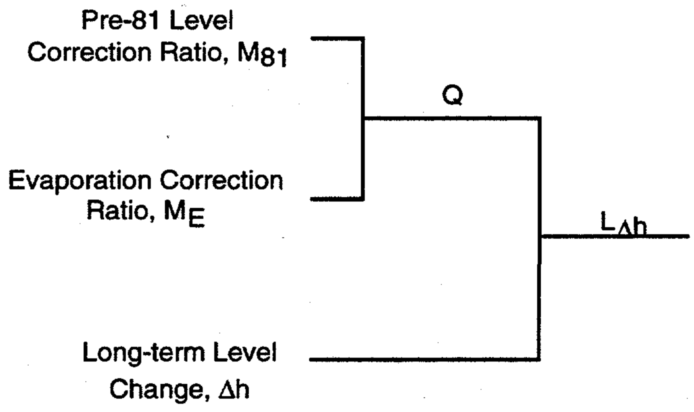

Fig. 3-7. Logic structure for determining long-term level change predictor likelihood, $L_{\Delta h}$.

*A more detailed model and a complete description of the basis for this approach is given in Hopkins (1994).

Correction ratios, M_E, M_{81} . The importance of the long-term level change estimate is affected by the magnitude of the estimates of the pre-1981 gas level change, Δh_{81}, and the evaporation change in relation to the measured level change since 1981 given by Δh_m. There can be wide variations in the quality of the estimates for both Δh_E and Δh_{81}, and if these parameters are the principal determinants of the long-term level change, the credibility of the estimate for Δh is reduced. The relative importance of Δh_E and Δh_{81} in determining Δh is represented in our model by the parameters M_{81} and M_E, which are defined as

$$M_{81} = |\Delta h_{81} / \Delta h_M| \tag{3-4}$$

and

$$M_E = |\Delta h_E / \Delta h_M| , \tag{3-5}$$

The larger the absolute value of these ratios, the larger is the influence of the poorly known parameters, Δh_E and Δh_{81}. The ratios M_{81} and M_E are represented as linguistic variables using the universe of discourse {{Small}, {Medium}, {Large}} ((M_{81} and $M_E \in$ {S,M,L}) where the descriptors refer to the size of the ratio. The quality estimate that is used to interpret the level change is based on these two ratios.

Table 3-9 summarizes the universes of discourse used for these parameters; the associated membership functions are shown in Fig. 3-8.

Implication Rule Bases for Long-Term Level Change Likelihood. The two ratios, M_{81} and M_E are combined to infer the quality parameter Q. The rule base for this inference is shown in Table 3-10. The quality is expressed using the sets {Poor}, {Fair} and {Good}, (Q$\in${P,F,G}. In this rule, if either M_{81} or M_E has membership in {Low}, then the quality parameter is reduced. This rule is almost symmetric, but slightly more weight is given to the ratio associated with the evaporation level estimate.

Table 3-9
Summary of Input Elements for Long-Term Level Change Logic

Parameter	Symbol	Universe of Discourse	Set Abbreviations	Rule Base Table	Membership Function Figure
Long-term Level Change	Δh	{{Very Small}, {Quite Small}, {Medium}, {Quite Large}, {Very Large}}	{VS,QS,M, QL,VL}	3-11	3-8
Evaporation Correction Term	M_E	{{Small},{Medium}, {Large}}	{S,M,L}	3-10	3-8
Pre-1981 Correction Term	M_E	{{Small},{Medium}, {Large}}	{S,M,L}	3-10	3-8

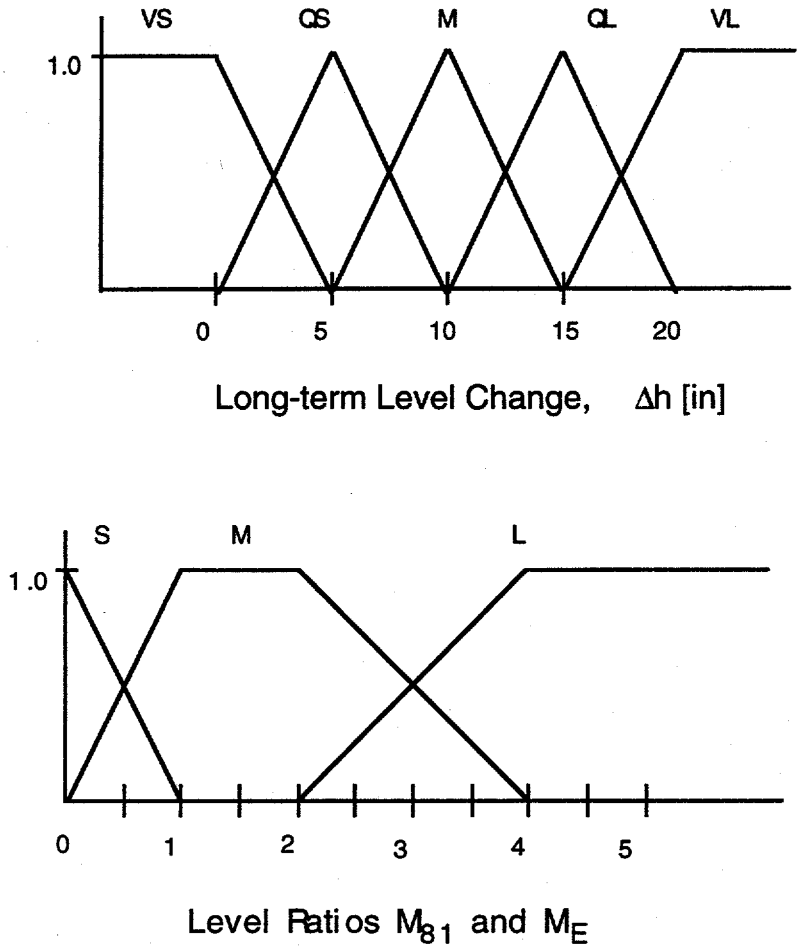

Fig. 3-8. Membership functions for Δh, M_{81} and M_E used in the $L_{\Delta h}$ logic submodule.

Table 3-10
Conflation Rule Base for M_{81} and M_E to Generate Q

Q Rules

M_{81}		M_E: S	M_E: M	M_E: L
	L	F	P	P
M_{81}	M	F	F	P
	S	G	F	P

The rule base for relating the level change, Δh, and the data quality modifier, Q, is shown in Table 3-11. The output of this conflation rule base implies a likelihood function, $L_{\Delta h}$, that is analogous to that for the barometric pressure measurements discussed in the previous section. If the quality is good, then the rule simply reflects the likelihood inferred directly from Δh. However, if the quality judgment is only "fair" or "poor," then the likelihood associated with Δh is relaxed toward membership in {Unresolved}.

Table 3-11
Conflation Rule Base for Δh and Q to Generate $L_{\Delta h}$

$L_{\Delta h}$ Rules

	G	VU	QU	U	QL	VL
Q	F	QU	U	U	U	QL
	P	U	U	U	U	U
		VS	QS	M	QL	VL
				Δh		

3.3.3. Aggregation of L_B and $L_{\Delta h}$ to obtain L_P. The aggregate predictor likelihood is inferred using the logic structure in Fig. 3-9. The rule base with L_B and $L_{\Delta h}$ as the antecedents is given in Table 3-12. This rule is symmetric in the weight given the two likelihoods, $L_{\Delta h}$ and L_B. The basic strategy of the rule is to intensify the output likelihood if the inputs agree. For example, a {Very Likely} and a {Quite Likely} combination of antecedents implies a {Very Likely} consequent for L_P, whereas a {Very Likely} and {Unresolved} set of antecedents implies {Quite Likely} for L_P. Mixed "unlikely" input memberships yield the mirror image of the "likely" ones . If the antecedents are contradictory, then the aggregate likelihood relaxes towards unresolved. For example, $L_{\Delta h} \in$ {Very Likely} and $L_B \in$ {Very Unlikely} results in $L_P \in$ {Unresolved}.

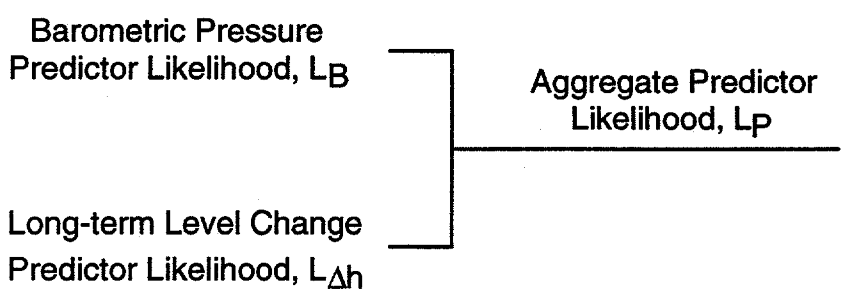

Fig. 3-9. Logic structure for combining barometric pressure and unexplained level predictors to obtain final predictor likelihood, L_P.

Table 3-12
Convolution Rule for Gas Predictors L_B and $L_{\Delta h}$ to Generate L_P

L_P Rules

	VL	U	U	VL	VL	VL
	QL	U	U	QL	VL	VL
L_{Dh}	U	VU	QU	U	QL	VL
	QU	VU	VU	QU	U	U
	VU	VU	VU	VU	U	U
		VU	QU	U	QL	VL
			L_B			

3.4. Enabler Likelihood Logic Module

Both gas generation and retention are required for a tank to pose a flammable gas hazard. In fact, the original FGWL criteria were based almost exclusively on waste characteristics and the degree of similarity to the waste in Tank SY-101. Certain tank characteristics act to enable gas generation or retention but are not sufficient alone to cause a hazard. These parameters are called gas enablers in this analysis. To obtain a likelihood based on enabler parameters, both gas generation and retention are examined. Parameters in the gas enabler category are similar to predictors in the sense that they can give indications of the presence of gas, but unlike predictors, they provide no direct information on the amount of gas. The overall logic structure for enabler likelihood, L_E, is shown in Fig. 3-10. There are two basic components of the gas enabler estimates—the potential for gas generation, G, and the potential for gas retention, R.

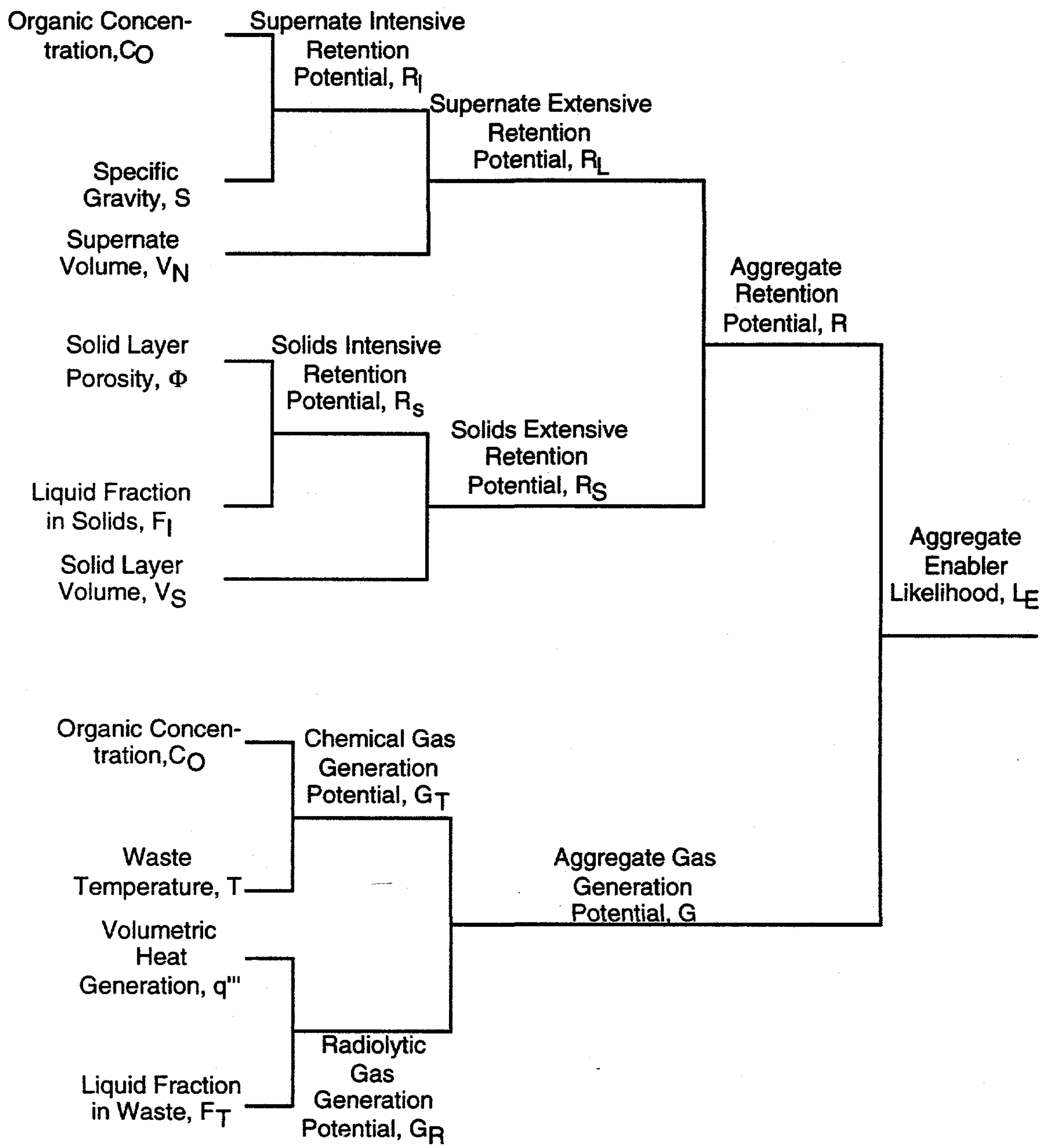

Fig. 3 -10. Overall logic tree for gas enabler likelihood, L_E.

3.4.1. Gas Generation Potential, G. Gas generation parameters are divided into thermolysis and radiolysis gas generation categories . Thermolysis occurs as a result of the thermal decomposition of organics in the waste. The following elements of evidence are used to draw inferences about thermolytic gas generation.

Total Organic Carbon, C_o. Generation of gas in the waste tanks has been identified strongly with the presence of complexants. In our model, this is represented by the primary input, C_o, the per cent of total organic carbon. The membership functions are based on an analysis by Agnew.* The universe of discourse for C_o is {{Low}, {Medium}. {High}} ($C_O \in$ {L,M,H}).

Waste Temperature, T. The other primary characteristic for chemical production of gas is the waste temperature. Membership functions used for waste temperature are intended for illustration only; more accurate membership functions can be developed based upon expert elicitation. We represent T linguistically with the sets {Low}, {Medium} and {High}, ($T \in$ {L,M,H}).

Radiolytic gas generation results from the decomposition of water, and the model logic assumes a dependence on the following elements.

Volumetric Heat Generation, q'''. In our simplified illustrative model, the gross radionuclide decay rate determines the radiolysis rate. This is just the known heat load divided by the waste volume. The universe of discourse for q''' is {{Low}. {Medium}, {High}} ($q''' \in$ {L,M,H}).

Liquid Fraction for Gas Generation, F_T. The other basic factor affecting gas production by radiolysis is the amount of available liquid. In the logic structure here, this is represented by F_T defined as

$$F_T = \frac{V_N + V_I}{V_T} , \tag{3-6}$$

where V is the volume of waste and the subscripts I, T, and N refer to interstitial liquid, total waste, and supernatant liquid, respectively. The interstitial liquid volume is found from

$$V_I = F_{SW} \Phi_C V_C + \Phi_D V_D , \tag{3-7}$$

where Φ and V are the porosity and volume of the salt cake (subscript C) and the sludge (subscript D) and F_{SW} is the fraction of salt cake that is wet. The sets describing F_T are {Low}, {Medium} and {High}, ($F_T \in$ {L,M,H}).

This module should be viewed as representative of more complex structures that could be developed to incorporate more detailed phenomenological pictures of waste behavior. The universes of discourse for these parameters and the rule bases in which they appear are given in Table 3-13; the corresponding membership functions are shown in Fig. 3-11.

Implication Rule Bases for Gas Generation Potential

In our logic model, the total organic content (TOC) in the waste is combined with the waste temperature to infer the gas generation activity from decomposition of organic waste components as shown in Fig. 3-12. A high temperature along with a high TOC provides the potential for high gas generation rates in the waste. This rule base (Table 3-14) is structured so that the influence of TOC is the stronger in determining generation potential and when the inputs are in agreement the output is intensified. The universe of discourse for G_T is {{Low}, {Medium}, {High}}, ($G_T \in$ {L,M,H}).

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*S. F. Agnew, "Correlation of FGWL Tanks with Total Organic Concentrations" Los Alamos National Laboratory, Private Communication (1996).

Table 3-13
Elements of Evidence Used to Infer Gas Generation Potential, G

Parameter	Symbol	Universe of Discourse	Set Abbreviations	Rule Base Tables	Membership Function Figure
Total Organic Carbon	C_O	{{Low},{Medium},{High}}	{L,M,H}	3-14	3-11
Waste Temperature	T	{{Low},{Medium},{High}}	{L,M,H}	3-14	3-11
Volumetric Heat Generation	q'''	{{Low},{Medium},{High}}	{L,M,H}	3-15	3-11
Liquid Fraction for Gas Generation	F_T	{{Low},{Medium},{High}}	{L,M,H}	3-15	3-11

Table 3-14
Conflation Rule Base for Combining C_o and T to Generate G_T

G_T Rules

C_o				
	H	M	H	H
	M	M	H	H
	L	L	L	L
		L	M	H

Temperature, T

The conflation rule base for combining q''' and F_T to yield a gas generation potential from radiolysis is shown in Table 3-15. In this rule, the volumetric heat generation rate has a stronger influence than does the liquid fraction in determining the generation potential from radiolysis. The universe of discourse for G_R is the same as for G_T.

Table 3-15
Conflation Rule Base for q''' and F_T to Generate G_R

G_R Rules

q'''				
	H	M	M	H
	M	L	M	M
	L	L	L	M
		L	M	H

Liquid Fraction, F_T

Fig. 3.11 (Quark Figure)

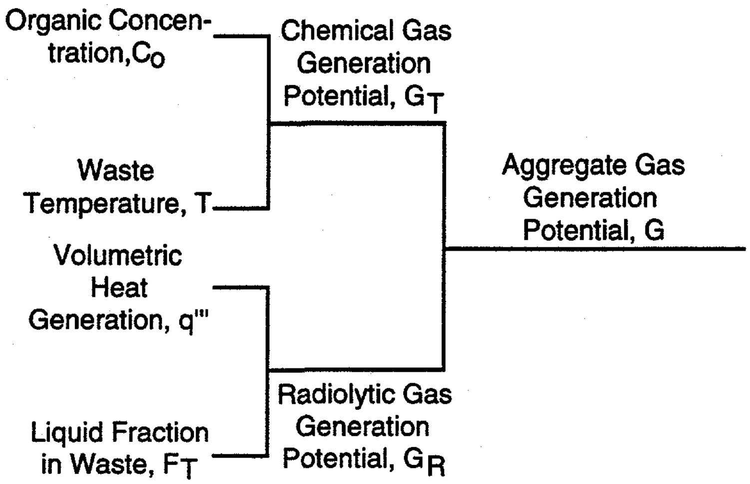

Fig. 3-12. Gas generation potential logic submodule.

The phenomenological rule base defining the relationship between the radiolysis and thermolytic generation potentials to infer the total gas generation potential, G, is shown in Table 3-16. G is expressed using the universe {{Low}, {Medium}, {High}, {Very High}}, ($G \in$ {L,M,H,VH}). Note that the output class of sets is expanded from the inputs. Also the influence of chemical production is given greater influence in this rule. If both potentials are high, then the rule evaluates to {Very High} for the total potential. The use of G to evaluate the complete enabler likelihood, L_E, will be discussed in Sec. 3.4.3 after the other component in its determination, the retention potential R, is considered.

Table 3-16
Conflation Rule Base for G_R and G_T to Generate Aggregate Generation Potential G

G

G_T \ G_R	L	M	H
H	H	H	VH
M	M	M	H
L	L	L	M

3.4.2. Gas Retention Potential, R. Gas retention is considered a strong function of the waste type, and this idea underlies the logic structure. Various retention mechanisms exist, and a more detailed logic structure would be needed to represent the level of complexity in this area. The pilot AR model deals only with an illustrative logic structure for evaluating gas trapped as bubbles.

The following parameters are used for determining supernate gas retention likelihood.

Total Organic Carbon, C_o. The TOC is known to be a factor in the retention of gas in slurry. The universe of discourse here is the same as for C_o as a primary input for chemical generation potential: {{Low}, {Medium} {High}} ($C_O \in$ {L,M,H}).

Specific Gravity, S. The solid fraction in a slurry has been related to the gas retention potential. Here we use the specific gravity as a measure of this property. Specific gravity is represented as a linguistic variable with the sets {Low}, {Medium}, and {High}, ($S \in$ {L,M,H}).

For salt cake and sludge waste types, the parameters used to determine waste retention effectiveness are as follows.

Solid Porosity, Φ. The solids layers need porosity to retain gas. The porosity used in this model is defined by

$$\Phi_{av} = \frac{\Phi_C V_C + \Phi_D V_D}{V_C + V_D}, \tag{3-8}$$

where Φ_i and V_i are the porosity and volume of the salt cake (subscript C) and sludge (subscript D), respectively. Thus, the porosity used is a waste volume (or height-)-averaged porosity. The universe of discourse is {{Low}, {Medium}, {High}}, ($\Phi \in$ {L,M,H}).

Liquid Fraction within Solid Layer, F_I. In our logic structure, it is assumed that gas retention within the pores will occur only if liquid is present. We use liquid fraction in the solid layers as a measure of this capability. The liquid fraction used for retention in the salt cake and sludge layers of the waste is defined by

$$F_I = \frac{V_I}{V_T - V_N}, \tag{3-9}$$

where again V_I is the volume of interstitial liquid, V_T is the total waste volume, and V_N is the volume of supernate. No distinction is made between salt cake and sludge. in the intrinsic gas retention capability The universe of discourse for F_I is {{Low}, {Medium}, {High}}, ($F_I \in$ {L,M,H}).

Total volume of supernate, V_N and solids, V_S. It is also necessary to account for the amount of material of a particular type available to retain gas. This is accomplished by using the total volume of supernate, V_N, and solids, V_S, as antecedents in conjunction with the intrinsic retention potential. Basically, consideration of the waste volumes allows one to extend inferences about the retention capability in waste types to a tank-specific basis.

The universe of discourses used to express these parameters as linguistic variables is given in Table 3-17; the corresponding membership functions for newly introduced elements of evidence are shown in Fig. 3-13

Table 3-17
Elements of Evidence Used to Infer Gas Retention Potential, R

Parameter	Symbol	Universe of Discourse	Set Abbreviations	Rule Base Table	Membership Function Figure
Total Organic Carbon	C_O	{{Low},{Medium}, {High}}	{L,M,H}	3-20	3-11
Specific Gravity	S	{{Low},{Medium}, {High}}	{L,M,H}	3-20	3-13
Total Volume of Supernate	V_N	{{Low},{Medium}, {High}}	{L,M,H}	3-21	3-13
Solid Porosity	Φ	{{Low},{Medium},{High}}	{L,M,H}	3-18	3-13
Liquid Fraction in Solid Layer	F_I	{{Low},{Medium},{High}}	{L,M,H}	3-18	3-13
Total Volume of Solids Layer	V_S	{{Low},{Medium}, {High}}	{L,M,H}	3-19	3-13

Fig. 3-13. (Quark figure)

Implication Rule Bases for Gas Retention Potential

The primary inputs for retention in liquid and solid layers are combined to yield a combined potential for gas retention, R, as shown in Fig. 3-14. Retention potential in solids layers is evaluated using the phenomenological rule base for Φ and F_l in Table 3-18. This is a symmetric rule in which both inputs are valued equally and the inferences are reflected about the diagonal. R_s is represented using the sets {Low}, {Medium}, and {High}, ($R_S \in$ {L,M,H}). This universe of discourse also is used for all subsequent inferences where some form of retention potential appears as the consequent.

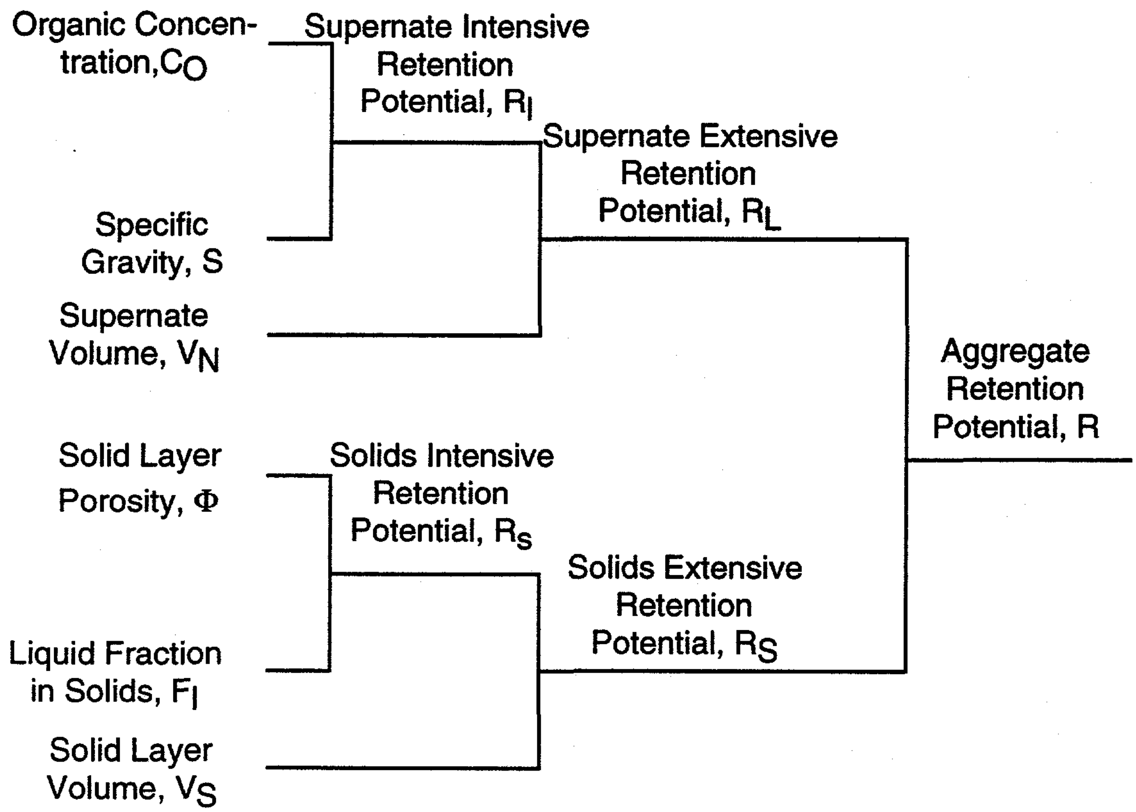

Fig. 3-14. Logic submodule for gas rentention potential.

Table 3-18
Conflation Rule for Φ and F_l to Generate R_s

R_S Rules

		Liquid Fraction F_l: L	M	H
Φ	H	L	M	H
	M	L	M	M
	L	L	L	L

The effect of total solids layer volume is accounted for with the rule base for R_s and V_S in Table 3-19. This rule base intensifies R_s if the waste volume is high.

Table 3-19
Conflation Rule Base for R_s and V_S to Generate R_S

R_S Rules

R_s				
	H	H	H	H
	M	M	M	H
	L	L	L	M
		L	M	H
			V_S	

Retention potential in the supernate layer is treated in the same manner, and the combination rules to obtain R_L are given in Tables 3-20 and 3-21.

Table 3-20
Conflation Rule Base for Combining C_o and S to Generate R_l

R_l Rules

C_o				
	H	M	H	H
	M	L	M	H
	L	L	L	M
		L	M	H
			Specific Gravity S	

Table 3-21
Conflation Rule Base for R_l and V_N to Generate R_L

R_L Rules

R_l				
	H	H	H	H
	M	M	M	H
	L	L	L	M
		L	M	H
			V_N	

In general, the waste in a tank may have both supernatant liquid and salt cake/sludge, so a rule for inferring the aggregate retention from the volume-averaged, waste-specific retention potential is given in Table 3-22. If the waste is judged to have only one waste type, then the retention of the other type is set to full membership in the {Low} set.

3.4.3. Aggregation of Generation and Retention Potential to Obtain the Enabler Likelihood, L_E. After the gas retention and generation potentials are evaluated, they are combined to infer the enabler gas likelihood, L_E (see Fig. 3-10). The implication rule base for this inference is shown in Table 3-23. As noted earlier, L_E is defined on the same universe of discourse as the predictor likelihood L_P – {{Very Unlikely}, {Quite Unlikely}, {Unresolved}, {Quite Likely} and {Very Likely}}, ($L_E \in$ {VU,QU,U,QL,VL}). Note that in the current rule base the generation potential is expressed with four sets and retention with only three. This assumes that one can express expert judgment about generation more precisely than for retention. Expert elicitation would be required to test this proposition. A symmetric rule using three sets to define G could be easily substituted.

Table 3-22
Conflation Rule Base for Different Waste-Type Retentions, R_L and R_S, to Generate the Aggregate Retention, Potential R

R Rules

R_L \ R_S	L	M	H
H	H	H	H
M	M	M	H
L	L	M	H

Table 3-23
Conflation Rule for Combining Generation, G, and Retention, R, to Give Enabler Likelihood, L_E

L_E Rules

R \ G	L	M	H	VH
H	U	QL	QL	VL
M	QU	QU	QL	QL
L	VU	VU	QU	U

The convolution of the enabler likelihood with those for the predictor and indicator likelihoods will be discussed in Sec. 3.6.

3.5. Indicator Likelihood Logic Module

The logic structures for both the predictor and enabler likelihoods were developed to facilitate judgments about evidence for gas retention in a tank. However for some tanks additional data are available related to observations of GRE. Our logic structure incorporates the fundamental inference that a GRE is evidence of gas retention. In certain tanks this evidence may be unambiguous, for example, in Tank SY-101 where dome-space measurements of hydrogen concentration above the LFL and coincident significant pressurization have been observed. In its unmitigated state SY-101 was the best example of a tank that exhibited Rayleigh-Taylor instability—the contents of the tank "rolled over" and evidence of this was apparent in level, pressure, and temperature changes. Although the evidence for other tanks is not necessarily so strong, the screening algorithm has been developed to include judgments based upon evidence of GRE behavior. This is done using a class of parameters referred to as gas indicators. Indicators can be positive or negative. When positive indicators are present they are very strong evidence of GRE behavior; a high dome-space flammable gas concentration measurement is an example. A negative indicator has the property that, if it is judged to be strongly present, the tank can be unambiguously excluded from the FGWL. For testing purposes we have used the WHC "Quick Screen" criteria (Hopson 1994) as a negative indicator. If a tank evidences this indicator then concentrations of flammable gas above the LFL in the tank dome space can be judged to be extremely unlikely. Each of the indicators is discussed separately below.

The complete logic structure for the class of gas indicator parameters is shown in Fig. 3-15. This is rather complicated but is intended to illustrate the many issues associated with inferring GRE behavior from the observational record. A condensation to the right-most chain of inferences is shown in Fig. 3.16. There are two separate groups of positive indicators for which likelihood judgments are made. The first uses the observational evidence from the dome space. The likelihood associated with this evidence is L_D. The second positive likelihood is L_W. It is used to evaluate the evidence of a GRE from observations of waste dynamics. All positive indicator likelihoods are defined on the same universe of discourse {{Unresolved}, {Quite Likely}, {Very Likely}, {Extremely Likely}} (L∈ {U,QL,VL,EL}).* "Extremely likely" is used only for indicators and expresses a higher level of certainty than one might associate with predictors or enablers. Membership in {EL} is meant to override mildly contradictory evidence from other parameters and indicate strongly the presence of gas. Finall,y the single negative indicator C_M uses the Quick Screen method to judge the worst-case gas concentration in the dome. In a more realistic implementation this primary input would be supplemented with other inputs to obtain a more broadly based negative indicator.

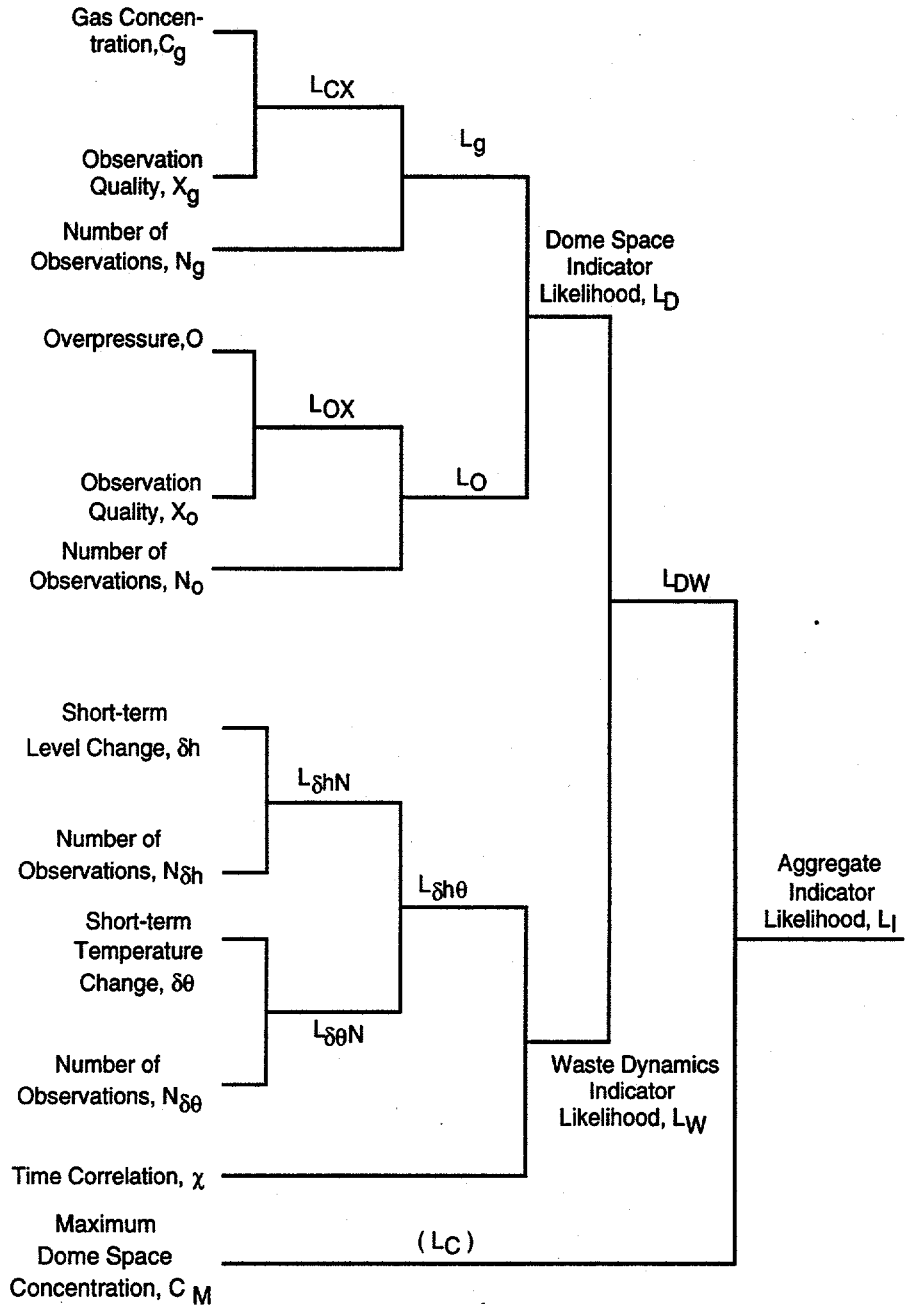

Fig. 3-15. Complete logic structure for gas indicator likelihood.

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*As noted earlier, positive indicators cannot disprove the existence of retained gas so the UOD for these likelihoods contain none of the unlikely hedges.

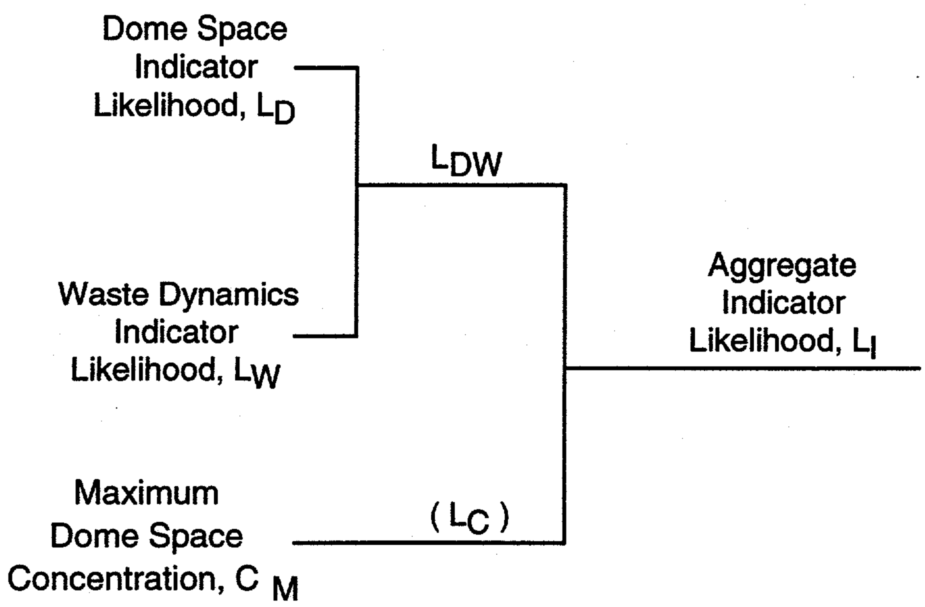


Fig. 3-16. Major component likelihoods for aggregate gas indicator likelihood, L_I.

3.5.1. Dome-Space Gas Indicator. Indication of a GRE using data from the dome space can be based on gas concentration measurements or upon evidence of dome pressurization. The two major likelihoods are L_g, the likelihood associated with the gas indicator based on dome-space concentration and L_O, a similar parameter for dome-space pressurization. The logic module for the dome-space indicator is shown in Fig. 3-17.

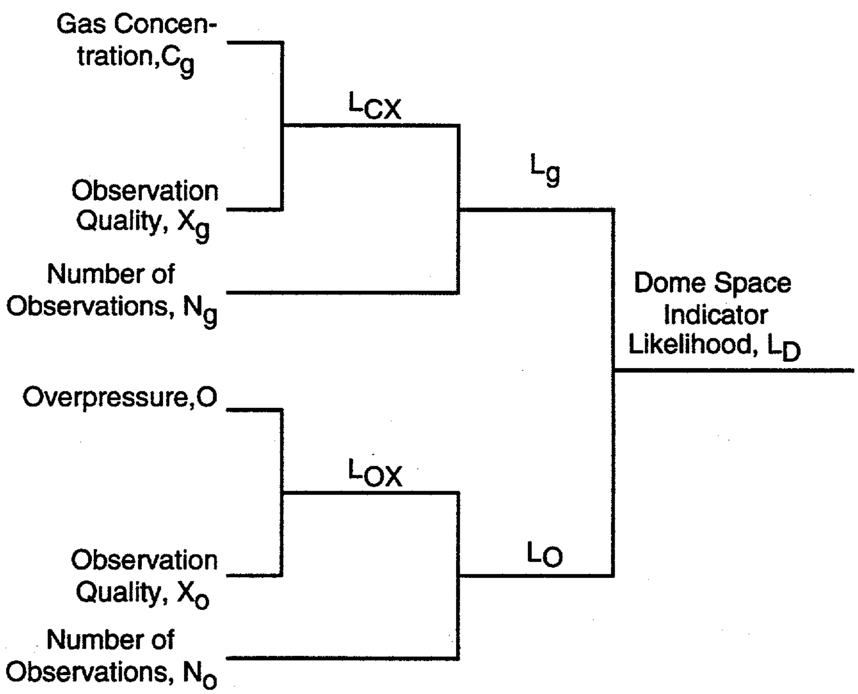


Fig. 3-17. Dome-space gas indicator combination rules tree.

The following elements of evidence are used in this submodule.

Dome-Space Concentration from GRE, C_g. A significant GRE would result in a sudden increase in the dome-space flammable gas concentration. If a concentration near the LFL is measured,* then a strong inference can be made that the tank retains a significant amount of gas. The only question remaining is to determine the validity of such gas-release measurements. A logic submodule could be developed for this evaluation if necessary. Judgments concerning instrument suitability and calibration as well as concerns about time-dependent tank behavior could be incorporated. We have chosen not to do this for the pilot logic structure here. Note if a steady-state gas concentration above the flammable limit were measured in a tank, it would be, by definition, a flammable gas tank. Thus, the steady-state gas concentration is, in principle, a gas indicator. However, all tanks recently were measured for steady-state flammable gas concentrations in the dome space, and none had a concentration close to the LFL. For this reason, the steady-state gas concentration was not included in the screening criteria discussed here. The gas concentration is expressed as {Low}, {Medium} and {High} ($C_g \in$ {L,M,H}), depending on its relationship to the LFL concentration. A high gas concentration is considered *prima facie* evidence of a GRE. Tank experts make the assignment of membership in the sets describing concentration directly based on their evaluation of measured flammable gas concentrations.

Evaluation of Observation Quality, X_g. An estimate of C_g may be based on any of several approaches to analyzing the observational data. Each of these approaches has some uncertainty or imprecision associated with it. To account for such factors, a quality parameter, X_g, is used. The function of X_g is to describe how the estimate was made. As an example of this approach, we express X_g using the sets {Measurement}, {Statistical Extrapolation}, or {Analytical Extrapolation}, ($X_g \in$ {M,SE,AE}). The set {Measurement} represents direct measurements of the maximum gas concentration. The set {Statistical Extrapolation} is for extrapolations from partial data that may not include the maximum concentration reached but provids a sound basis for determining a time-dependent concentration profile. The set {Analytical Extrapolation} is for situations where fragmentary data are used with a model of gas release and transport to estimate the maximum gas concentration. These sets are fuzzy, so membership in more than one set for X_g is allowed.

Number of Observations, N_g. The number of observed events is also important in determining the quality of the concentration event data. A single or rare event has less weight and credibility than a large number of events. The variable N_g is a measure of the number of potential events that have occurred and can have membership in sets {Few}, {Several} or {Many}, ($N_g \in$ {F,S,M}).

Dome-Space Pressurization from GRE, O. A gas-release event can pressurize the dome space suddenly. The measure used for tank pressurization events could be, for example, the sample mean of observed overpressure events. The magnitude of the event is an indication of the size of the GRE and hence retained gas.** The question remaining is to determine the validity of such measurements, and the discussion above concerning concentration measurements applies here as well. Pressurization values use the universe {{Low},{Medium},{High}}, ($O \in$ {L,M,H}).

Evaluation of Observation Quality, X_O. The basis for the estimate of O is assessed using sets that indicate the type of judgment used in determining the magnitude of the overpressurization. The quality input here is of exactly the same form as for X_g. The primary input, O, is modified by measures of certainty given by X_O, with the universe of discourse of {{Measurement}, {Statistical

* We ignore here the question of computing the lower flammability limit for an uncertain mixture of gases. Note however that this is less critical when using fuzzy sets to represent the concentration linguistically than when a sharp threshold is used.

** We neglect here the question of release fraction.

Extrapolation}, {Analytical Extrapolation}}, ($X_O \in$ {M,SE,AE}). The use of this universe of discourse is the same as for the observation quality for gas concentration discussed above.

Number of Observations, N_O. The number of observed events is important in determining the quality of the pressure event data. A single or rare event has less weight and credibility than a large number of events. The variable N_O is a measure of the number of potential events that have occurred and can have membership in sets {Few}, {Several} or {Many}, ($N_O \in$ {F,S,M}).

Table 3-24 gives the universes of discourse for these elements of evidence, and Fig. 3-18 gives the corresponding membership functions. The magnitudes used here for overpressurization are for illustration purposes. In practice, separate membership functions for different tank sizes or a more complicated logic structure that incorporates dome-space volume may be necessary.

Table 3-24
Elements of Evidence used to Infer Dome-Space Indicator Likelihood, L_D

Parameter	Symbol	Universe of Discourse	Set Abbreviations	Rule Base Table	Membership Function Figure
Dome-Space Gas Concentration	C_g	{{Low},{Medium}, {High}}	{L,M,H}	3-25	3-18
Dome Overpressure	O	{{Low},{Medium}, {High}}	{L,M,H}	3-25	3-18
Concentration or Pressure Observation Quality	X_g X_O	{{Measurement}, {Statistical Extrapolation}, {Analytical Extrapolation}}	{M,SE,AE}	3-25	NA
Number of Concentration or Overpressure Observations	N_g N_O	{{Few},{Several}, {Many}}	{F,S,M}	3-26	3-18

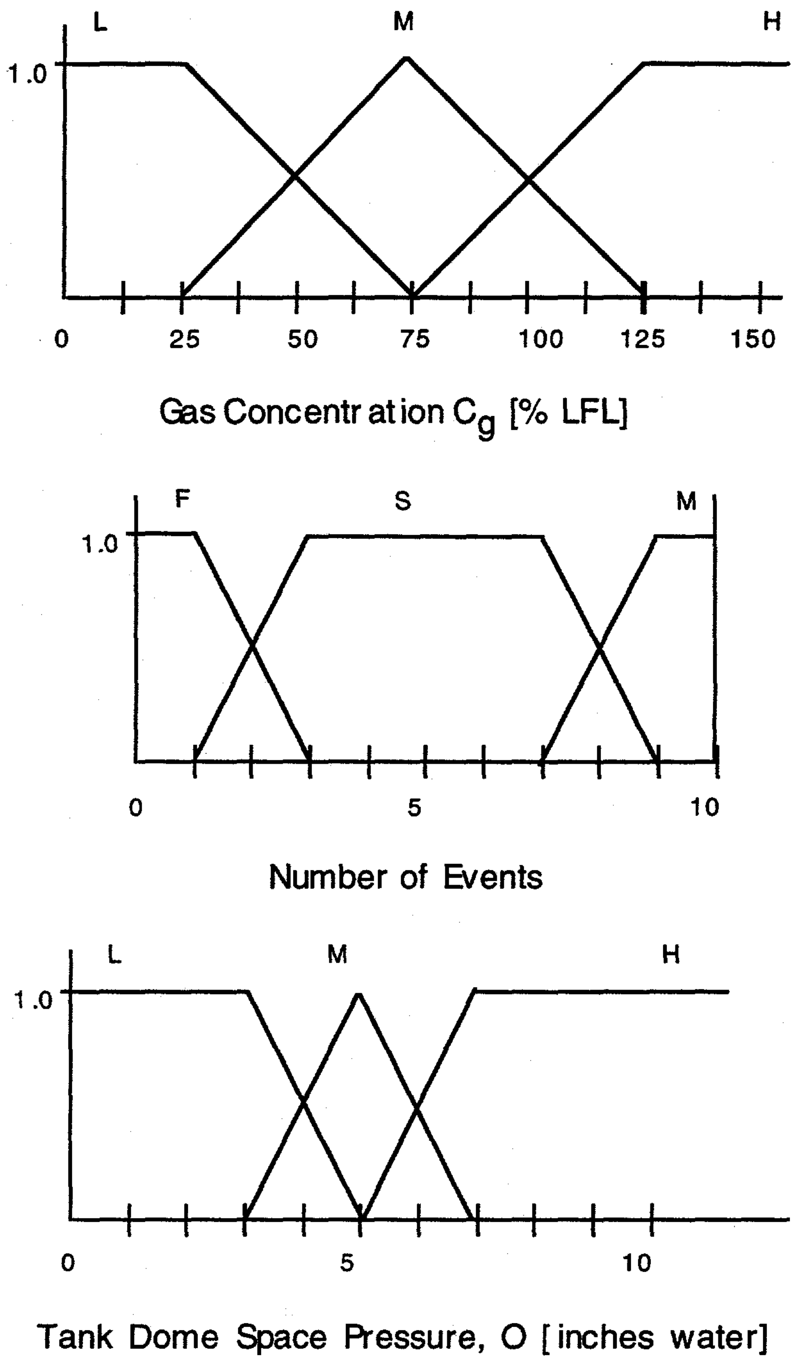


Fig. 3-18. Membership function for input elements in the dome-space indicator logic submodule.

Implication Rule Bases for Dome -Space Indicator Likelihood

Figure 3-19 shows the logic submodule for inferring the dome-space gas concentration likelihood, L_g. The concentration, C_g, and the certainty, X_g, are combined using the rule base in Table 3-25 to generate a concentration gas likelihood, L_{CX}.

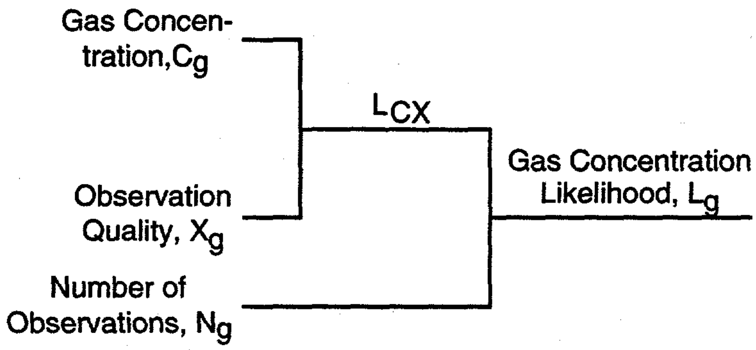


Fig. 3-19 Logic submodule for dome-space gas concentration likelihood.

Table 3-25
Conflation Rule Base for C_g and X_g to Generate Likelihood L_{CX}

L_{CX} Rules

C_g				
	M	U	QL	EL
	SE	U	U	VL
	AE	U	U	QL
		L	M	H
			C_g	

The output of this rule is combined with the number of events that have occurred, N_g, to generate the dome-space concentration likelihood, L_g, using the rule base shown in Table 3-26. Here the number of measurements, N_g, acts as a qualifier on the likelihood implied by the concentration, C_g. A high measured dome-space gas concentration likelihood, $L_{CX} \in$ {Extremely Likely} implies a high likelihood of retained gas, so $L_g \in$ {Extremely Likely} if several or many events have been observed, but with membership only in {Quite Likely} possible when there are few measurements. Membership in {Unresolved} for L_{CX} implies that a GRE has probably not been observed. This leaves the evaluation unresolved rather than implying that gas retention is unlikely. Fewer measurements always relaxes the likelihood implied by L_{CX} for many measurements toward membership in {Unresolved}. The same rule bases are used to infer L_O with O substituted for C_g, X_o for X_g, and N_O for N_g.

Table 3-26
Conflation Rule Base for Combining L_{CX} and N_g to Generate L_g

L_g Rules

L_{CX}				
	EL	QL	EL	EL
	VL	U	VL	EL
	QL	U	QL	VL
	U	U	U	U
		F	S	M
			N_g	

The indicator likelihoods L_g and L_O for concentration and pressurization evaluations are used as antecedents in the rule base for the overall dome-space indicator likelihood L_D. The rule is shown in Table 3-27. This rule incorporates strong intensification and relaxation to obtain either a definitive or an unresolved judgment. In this rule, if either antecedent likelihood is {Extremely Likely}, then so is L_D. Note that L_D is unresolved if one antecedent is unresolved and the other is either unresolved or quite likely. L_D appears as an input to the final indicator likelihood, L_I. This is discussed in Sec. 3.5.4

Table 3-27
Convolution Rule Base for L_g and L_O to Create L_D

L_D Rules

L_g \ L_O	U	QL	VL	EL
EL	EL	EL	EL	EL
VL	QL	VL	EL	EL
QL	U	QL	VL	EL
U	U	U	QL	EL

3.5.2. Waste Dynamic Indicator Likelihood Logic Submodule. Sudden changes in tank level or in the waste temperature profile are strong indications of a GRE. A logic structure based on this assertion is shown in Fig. 3-20. Indicator likelihoods based upon short-term level change and short-term changes in tank axial temperature profile are used. When either of these likelihoods is large, a GRE is strongly indicated. If there is a multiplicity of such events occurring periodically and correlated in time, then one expects that the indication would be even stronger. This logic is implemented by including a primary input χ to evaluate the degree of correlation between short-term level and temperature changes.

Long-term level change was discussed as a gas predictor. A sudden change in level can occur during a GRE. The relationship between the long-term and short-term level changes as used in this study is shown in Fig. 3-21. Larger short-term level drops are indicative of larger release events. Gross material motion is also observed in some GREs, so changes in temperature profile are also evidence considered in this submodule. The evaluation depends simply on the size of the short-term level or temperature changes, some judgment of their observed frequency, and whether the two signals are correlated.

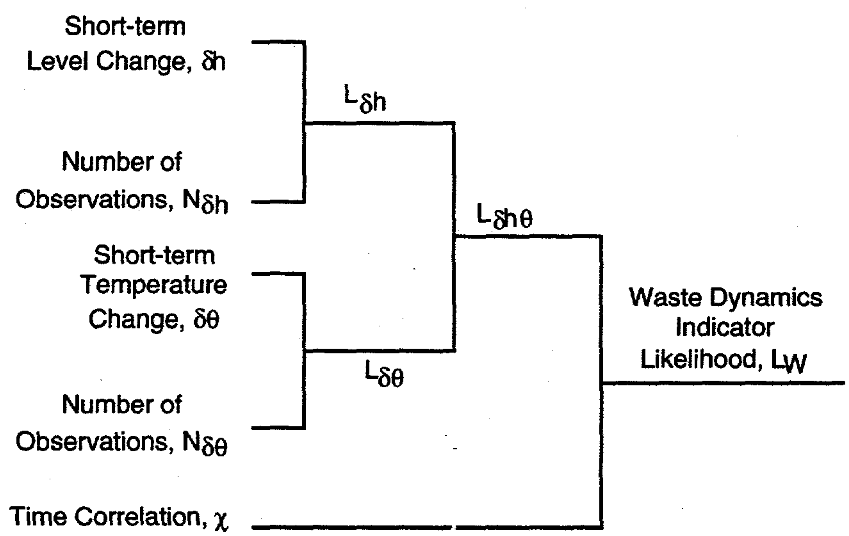


Fig. 3-20. Logic structure for waste dynamics likelihood.

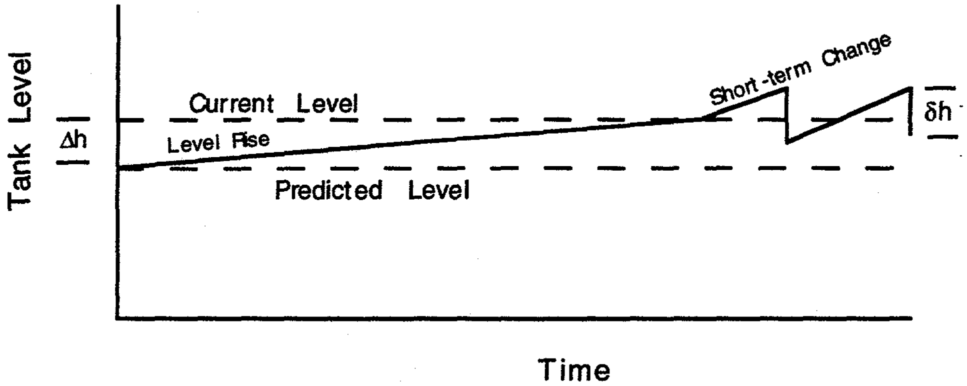


Fig. 3-21. Level parameters used in the screening algorithm.

The following elements of evidence are used in this submodule.

Magnitude of Short-Term Level Change, δh. Expert judgment is used to evaluate short-term level changes. The experts make their judgment on the size of such changes based on the following criteria.

1. Have one or more rapid changes in level occurred in the tank?
2. What was the characteristic magnitude of the events?
3. How confident are they that the event did occur?
4. Are the indications and instruments redundant?

The characteristic level change for the events is represented with the universe of discourse of {{Small}, {Medium}, {Large}}, ($\delta h \in \{S,M,L\}$).

Periodicity for Short-term Level Change, $N_{\delta h}$. Expert judgment also is used to evaluate the temporal characteristics of the short-term level changes. The experts make their judgment of such changes using a criterion such as the following.

Is there a history or multiplicity of such events or does the event occur periodically?

The number $N_{\delta h}$ is represented linguistically with the sets {Isolated}, {Sporadic} or {Periodic}, ($N_{\delta h} \in \{I,S,P\}$). Isolated means a single or a very few events; sporadic means events at irregular intervals; periodic suggests regularly occurring events in considerable numbers.

Short-term temperature Change, δθ. The temperature profile change, δθ, is a qualitative judgment and is assigned membership directly in the groups {Unlike}, {Similar} and {Identical}, ($\delta\theta \in \{UL,SM,ID\}$) that describe how closely the observed temperature profile change agrees with that historically associated with a GRE. Membership in more than one set is possible.

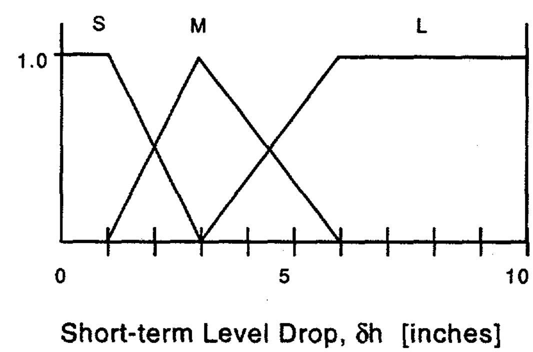
Periodicity for Short-term Temperature Change, $N_{\delta\theta}$. The temporal characteristics of short-term temperature changes are evaluated using expert judgment. The experts make their judgment of such changes using the same criteria and universe of discourse as for level change.

Waste Dynamic Parameter Correlation Parameter, χ. If the changes in level and temperature are correlated in time, then the waste dynamic indicator would be considered to be stronger. The degree of correlation is evaluated using expert judgment. The time correlation is expressed by the universe of discourse of {{Low}, {Medium}, {High}}, ($\chi \in \{L,M,H\}$). The experts rank the correlation, χ, of level and temperature changes according to a numerical scale from 0 to 1 with 1 meaning perfect temporal correlation.

Table 3-28 gives the universes of discourse and where they appear in rule bases for these elements of evidence, and Fig. 3-22 gives the corresponding membership functions. Note that for several of these elements there are no membership functions specified. The DOMs for these qualitative elements are assigned directly based on expert judgment.

Table 3-28
Elements of Evidence Used to Infer Waste Dynamics Indicator Likelihood, L_W

Parameter	Symbol	Universe of Discourse	Set Abbreviations	Rule Base Table	Membership Function Figure
Short-Term Level Change	δh	{{Small},{Medium}, {Large}}	{S,M,L}	3-29	3-22
Periodicity for Short-term Level Change	$N_{\delta h}$	{{Isolated}, {Sporadic}, {Periodic}}	{I,S,P}	3-29	NA
Short-term Temperature Change	$\delta\theta$	{{Unlike},{Similar}, {Identical}}	{UL,S,I}	3-30	NA
Periodicity for Short-term Temperature Change	$N_{\delta\theta}$	{{Isolated}, {Sporadic}, {Periodic}}	{I,S,P}	3-30	NA
Waste Dynamics Correlation	χ	{{Low},{Medium}, {High}}	{L,M,H}	3-32	3-22



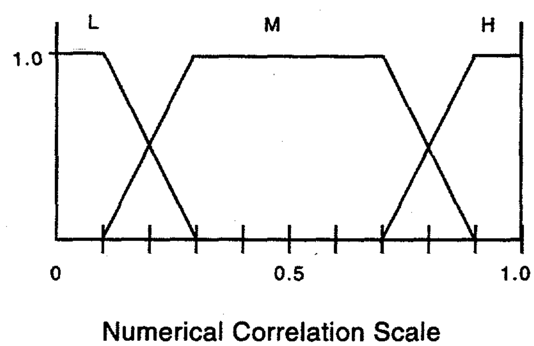


Fig. 3-22. Membership functions for input elements in the waste dynamics indicator module.

Implication Rule Bases for Waste Dynamics Likelihood

The level drop, δh, is combined with $N_{\delta h}$ to imply a likelihood, $L_{\delta h}$, according to the rule base shown in Table 3-29. Note that this rule only implies membership in {Extremely Likely} if the level changes are large and if they are periodic.

Table 3-29
Qualification Rule Base for dh AND N_{dh} to Give L_{dh}

$L_{\delta h}$ Rules

δh \ $N_{\delta h}$	I	S	P
L	QL	VL	EL
M	U	QL	VL
S	U	U	QL

The rule used to evaluate L_θ based on the temperature changes and their temporal characteristics is given in Table 3-30. This rule is identical in structure to that for $L_{\delta h}$.

Table 3-30
Qualification Rule Base for dq and N_q to Give L_q

L_θ Rules

$\delta\theta$ \ $N_{\delta\theta}$	I	S	P
I	QL	VL	EL
S	U	QL	VL
UL	U	U	QL

The temperature and level change likelihoods L_θ and $L_{\delta h}$ are antecedents for the likelihood rule base given in Table 3-31 to infer an intermediate likelihood, $L_{\delta h\theta}$. Here the implication operator uses strong intensification and relaxation to differentiate between tanks where the waste dynamics indicator data are clear and where they are ambiguous.

Table 3-31
Convolution Rule for L_θ and $L_{\delta h}$ to Create $L_{\delta h\theta}$

$L_{\delta h\theta}$ Rules

L_θ \ $L_{\delta h}$	U	QL	VL	EL
EL	EL	EL	EL	EL
VL	QL	VL	EL	EL
QL	U	QL	VL	EL
U	U	U	QL	EL

The rule for evaluating the likelihood, L_W, based on the expert judgment on the appropriate interpretation of the short-term changes and the strength of the time correlation between the level and temperature changes, is shown in Table 3-32. A high value of χ intensifies the implication.

Table 3-32
Qualification Rule for $L_{\delta h\theta}$ and χ to Create L_W

L_W Rules

	H	U	VL	EL	EL
χ	M	U	QL	VL	EL
	L	U	U	QL	EL
		U	QL	VL	
			$L_{\delta h\theta}$		

3.5.3. Logic Submodule for Maximum Dome-Space Concentration from Quick Screen, C_M. The dome space and waste dynamics indicators are both positive. That is, if they evaluate to a likelihood of extremely likely then the evidence for flammable gas behavior is very strong. By the same token it would be useful to have a negative indicator. A negative indicator has a threshold value that indicates conclusively that some necessary condition for flammable gas concentrations in the dome space—gas generation, retention and release or sufficient concentration of flammable gas *in situ*—is not possible in the tank because of some physical characteristics of the waste/tank environment. As a model for this class of indicators, we have used the maximum dome-space concentration, C_M, obtained from the WHC quick-screen methodology (Hopkins 1995). As used here,

$$C_M = \alpha V_S \; , \qquad (3\text{-}10)$$

where α is a tank-specific parameter with units per cent LFL/kilogallon of waste and V_S is the volume of waste in the solids layer. The constant α is derived from data for Tank SY-101 and is used to estimate the maximum amount of gas that could be released from the waste. This gas is assumed to be 97% hydrogen. The key idea is that if a tank passes this extremely conservative test, then it is incapable of being a FGWL tank. The universe of discourse for C_M is {{Very Low},{Low},{Medium},{High}}, ($C_M \in$ {VL,L,M,H}). The membership functions for C_M are shown in Fig. 3-23. A high DOM in {VL} should preclude the tank being placed on the FGWL. Membership in {L} will support other negative likelihoods.

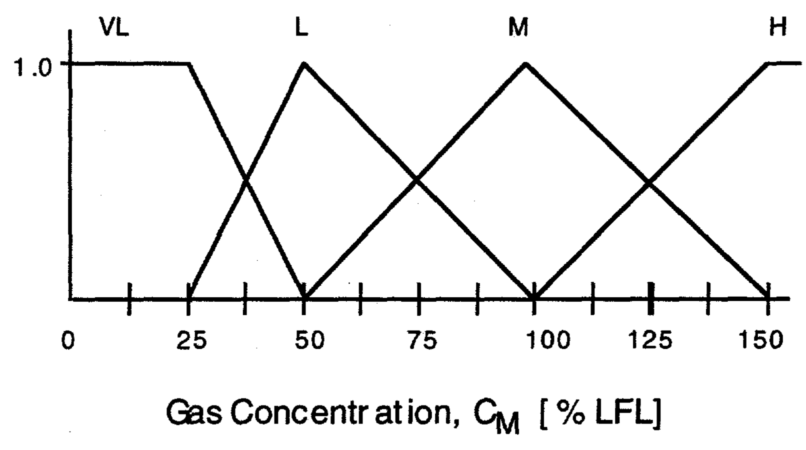


Fig. 3-23. Negative indicator gas likelihood membership function, L_C.

3.5.4. Conflation of L_D, L_W, and C_M to Obtain the Aggregate Indicator Likelihood. The evaluation of L_I involves logic rules for combining the dome-space indicator likelihood, L_D; the waste dynamics indicator, L_W; and the maximum gas concentration, C_M. This is shown in Fig. 3-24. We choose to use the two positive indicators as antecedents to obtain an intermediate positive GRE indicator likelihood, L_{DW}. The rule base for this inference is shown in Table 3-33. Note that good agreement between the two likelihoods intensifies the judgment of the likelihood of a GRE.

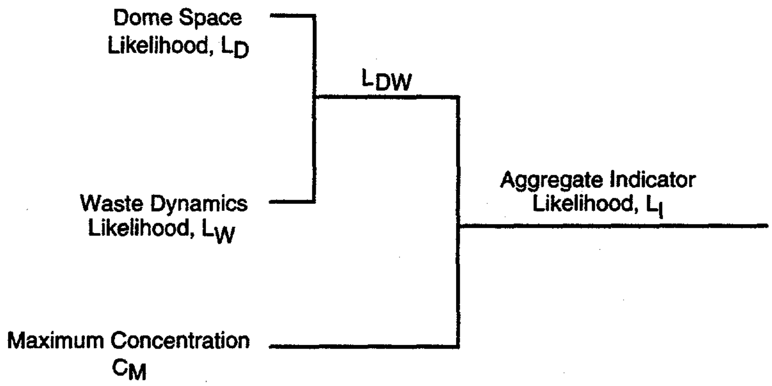


Fig. 3-24 Logic structure for evaluating L_I.

Table 3-33
Convolution Rule for L_D and L_W to Create L_{DW}

L_{DW} Rules

	EL	EL	EL	EL	EL
	VL	QL	VL	EL	EL
L_D	QL	U	QL	VL	EL
	U	U	U	QL	EL
		U	QL	VL	EL

L_W

The GRE and the maximum concentration likelihoods then are combined to infer an aggregate gas indicator likelihood, L_I. The rule for combining these parameters is shown in Table 3-34. This rule is quite different from the one above. This is because we are evaluating two parameters where a high DOM in {Extremely Likely} for the positive indicators and membership in {Very Low} for C_M is a physical and logical contradiction. In this case, we ought to assume that some aspect of the primary input data is incorrect and judge the indicator expectation to be unresolved. On the other hand, if one parameter has a strong membership in an extreme set and the other is unresolved, then a strong judgment is possible. Other versions of this rule base could be used if it were preferable to weigh the positive and negative indicators differently.

Table 3-34
Rule Base for Combining L_{DW} and C_M to Create L_I

L_I Rules

C_M					
	VL	EU	U	U	U
	L	U	U	U	U
C_M	M	U	U	U	U
	H	U	U	U	EL
		U	QL	VL	EL
			L_{DW}		

3.6. Aggregation of Predictor, Enabler, and Indicator Likelihoods to Obtain the Evaluation Output Likelihood, L_F

The likelihood evaluations from the three major likelihood classes, L_P, L_E, and L_I, act as antecedents to infer the final aggregate gas retention likelihood, L_F. (See Fig. 3-1.) The linguistic variable L_F is the last output from the algorithm and provides the basis for the decision whether to screen. We consider the indicator evaluation to be distinctly different from the predictor and enabler evaluations and therefore chose to develop an intermediate inference first with the latter two as antecedents. Note that this is a judgment built into the structure of the model. We could have chosen to combine these likelihoods in a different order or, alternatively, to have constructed a single rule base with all three likelihoods as direct antecedents for L_F. The rule base for combining the gas predictor and gas enabler likelihoods is shown in Table 3-35. The intermediate positive indicator likelihood has the same universe of discourse as do L_P and L_E.

Table 3-35
Convolution Rule for Predictor L_P and Enabler L_E to Create L_{PE}

L_{PE} Rules

	VL	U	U	VL	VL	VL
	QL	U	U	QL	VL	VL
L_P	U	VU	QU	U	QL	VL
	QU	VU	VU	QU	U	U
	VU	VU	VU	VU	U	U
		VU	QU	U	QL	VL
			L_E			

The rule base used for chaining this inference with the aggregate indicator likelihood is shown in Table 3-36. Note that if $L_I \in \{\text{Unresolved}\}$, then the DOM in L_{PE} is reflected—the consequent, L_F, has the same DOMs as does L_{PE}, $\mu(L_F) = \mu(L_{PE})$. If the antecedent likelihoods are in agreement, then the judgment of L_I is the output. This is consistent with the power assigned to the aggregate indicator likelihood in the model rules provided above.

Table 3-36
Convolution Rule for Combining L_I and L_{PE} to Create L_F

L_F Rules

L_I \ L_{PE}	VU	QU	U	QL	VL
EL	U	U	EL	EL	EL
U	VU	QU	U	QL	VL
EU	EU	EU	EU	U	U

L_{PE}

3.7. Probabilistic Expression for L_F

As discussed briefly in Sec. 2, the aggregate likelihood, L_F, is a random variable. This is true because most of the inputs to the inductive logic structure are themselves random. The inescapable uncertainty in L_F means that any useful statement about the aggregate likelihood will be statistical in nature. We wish to express L_F in the same terms as the likelihood sets that constitute its universe of discourse. This means that a mechanism for measuring L_F quantitatively is needed to compute the appropriate statistics and, second, that an additional operation must be used to express the statistical properties of L_F as natural language expressions.

3.7.1. Probability Density Functions for Input Elements of Evidence. There is no practical way to determine the probability density function (PDF) for L_F directly from the input parameters' PDFs because the total number of inputs is large and because of the nonlinear min-max operations performed for each implication rule base. Therefore, the statistics for L_F must be obtained from Monte Carlo (MC) sampling. The MC simulation consists of N trials where, for each trial, all of the input parameters are sampled from their defining PDFs and a complete evaluation is performed with these sample inputs. The immediate output from each trial is an estimate for L_F that is a DOM vector. We consider below how input PDFs are defined, and then we discuss the calculation of statistics for L_F and the procedure used to transform these statistics into a natural language expression for the aggregate likelihood. This natural language expression then is compared with a criterion to determine whether the tank fails or passes the screen or whether the evidence leads only to an unresolved conclusion.

For quantitative input parameters, the use of PDFs in the MC simulation here is very similar to that in many applications. That is, for each trial i, the value of an input x_i is given by x_i = PDF (density, parameters, seed, i) where density is the particular form of PDF used to represent x, parameters are the set of numerical values required to specify the exact density function, seed is the number used to start generating a sequence of random numbers, and i denotes the i^{th} value of x generated. Quite often, a PDF is specified by either the mean, x_m, or median, x_{50} and the standard deviation, σ. In many purely numerical algorithms, the PDF is used primarily to represent epistemic uncertainty. In such cases, there is a tendency to use large values of σ to "cover all the bases" and ensure that the tails of the distribution have some detectable influence on the final output. This approach is unnecessary and is to be avoided in AR models. The use of implication rule bases allows one to make explicit qualitative judgments about the quantitative aspects of data; hence, there is no need to increase the variance to roughly approximate such considerations.

A qualitative input also may be uncertain. In this case, we need some method to assign DOMs in the sets used to express its qualitative value. We encountered this situation in Sec. 2 when the problem of describing the temperature in a room without using a thermometer was discussed. Two simple approaches that are consistent with how experts often describe qualitative data are used in this report. In the first of these, we introduce a numerical scale to convert a linguistic variable to a quantitative value and define membership functions for this converted variable to obtain DOMs. For example, this approach was taken with χ, the waste dynamics correlation factor, which is defined on the interval 0 to 1. An assignment of $\chi = 0.9$ means that the level and temperature changes associated with a GRE are considered to be well-correlated; it corresponds to full membership in the set {High}. The expert or group of experts provides input for the specification of a PDF for χ in the range [0,1] that is completely analogous to that for a quantitative input. In the second approach, used here more often, DOMs for a qualitative input are assigned directly. This could be done either by the expert himself or by an elicitor who interprets the expert's statements about the variability he associates with the input and translates it into an appropriate PDF.

A complete evaluation with all of the major logic branches described here requires the specification of 40 parameters needed to calculate the primary inputs to the logic structure. In general, the PDFs are taken from Hodgson (1995). For some parameters, variation of the parameter depends on other inputs. For example, this is true of C_M, which is a function of the total volume of waste in the solids layer. It also should be noted that the heights of the waste layers and the total volume of waste are treated as dependent variables. In these cases, the mean is taken from Hodgson, but the variation is obtained from a simple auxiliary equation. For example, the height of the sludge layer is defined by $h_D = \beta V_D$, where β is the ratio of the means of the height and layer volume.* This approach ensures that all correlated quantities are treated correctly in the MC simulations. All of the qualitative input parameters appear in the rules used to judge whether a tank exhibits GRE behavior. That is, in the positive portion of the indicator likelihood module. For testing purposes, these parameters were specified so that there would be no clear evidence of GRE behavior. This is consistent with the data available for the tanks used to demonstrate the AR methodology. In actual use, it would be necessary to have a group of experts supply their judgment about these qualitative factors.

3.7.2. Statistical Measures for L_F. The DOMs for L_F in the sets {{Extremely Unlikely}, {Very Unlikely}, {Quite Unlikely}, {Unresolved}, {Quite Likely}, {Very Likely}, {Extremely Likely}} are computed at the conclusion of each MC trial. We denote this as

$$L_{Fi} = \gamma_i\,(EU,VU,QU,U,QL,VL,EL) = [\gamma_i\,(S_j)\; j = 1,7]\ , \tag{3-11}$$

where $\gamma(S_j)$ is the DOM in set j. At the end of the simulation, we have N estimates for L_F. There are two ways to calculate statistics for L_F using this vector.

It can be seen that there are seven distinct density function estimates associated with L_F:

$$PDF(L_F) = [\ PDF(\gamma(S_j)\; j = 1,7)]\ . \tag{3-12}$$

One approach is to derive the statistics from this vector. In this case, if we ask about the value of L_F at some quantile, q_i, associated with $PDF(L_F)$ we use the vector

$$q_i^* = [\ q_i(\gamma(S_j)\; j = 1,7)]\ . \tag{3-13}$$

Thus, the vector contains the DOMs at the q_i quantile for each set in the universe of discourse for L_F. However, note that the vector q^* is not itself the q_i quantile for L_F. We must specify how to process the vector to compute $q_i(L_F)$. A natural approach is to define $q_i(L_F)$ as

$$q_i(L_F) = \max\,[\ q_i(\gamma(S_j)\; j = 1,7)]\ . \tag{3-14}$$

This specification for q_i is the maximum DOM associated with the likelihood sets at this quantile.

---

*In theory, β only depends on the cross-sectional area of the tank. However, the best-estimates for volume and waste height do not always satisfy this relationship. Therefore, β is calculated from the volume and height estimates.

A second approach to estimating statistics for L_F involves calculating a measure from L_{Fi} for each trial and then obtaining an estimate for the density function of this measure in the MC simulation. The process of calculating a single measure from DOMs in a class of fuzzy sets is called defuzzification. To defuzzify, it is necessary to define membership functions for L_F. These are shown in Fig. 3-25. The asymmetry in the membership functions here is intended to illustrate how degree-of-conservatism considerations can be incorporated into an AR model. In this case, the membership functions reflect an adversion to classifying the aggregate likelihood as either extremely or very unlikely. The choice of membership functions will be discussed further in Sec. 5.

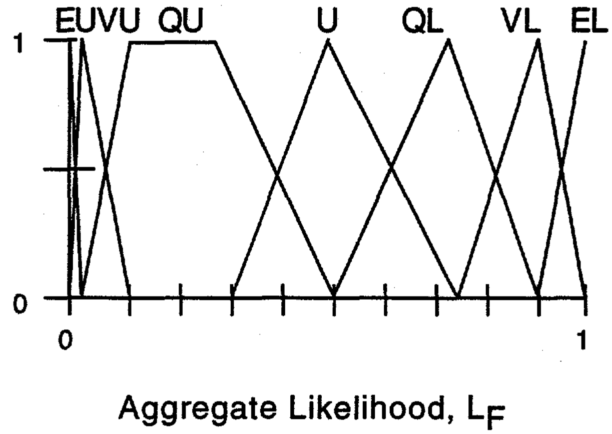


Fig. 3-25 Membership functions for L_F.

A common defuzzification measure is the centroid. In this approach, each membership function is multiplied by the actual DOM. We denote these weighted functions as C_j. The union of the C_j functions,

$$\bigcup_j C_j ,$$

is obtained using the normal max operator. This yields the outer envelope in regions where the C_j's overlap. The centroid λ for L_F resulting from a single MC trial is

$$\lambda_i(L_F) = \int_0^1 x \bigcup_j C_j \, dx \Big/ \int_0^1 \bigcup_j C_j \, dx \quad (3\text{-}15)$$

The centroid normally is considered to be the best-estimate approach to defuzzification; many other estimators exist (Ross 1995).

As an example of centroid defuzzification, consider L_{Fi} = [0,0,.1,.6,.4,.2,0]. Figure 3-26 shows the DOM-weighted union. In this case, the centroid is $\lambda_i(L_F)$ = 0.56. After N MC trials, the PDF for the centroid can be estimated, and the desired statistical properties can be obtained. We denote the q_i quantile of the centroid by $\lambda_{LF}(q_i)$.*

*Clearly, one can also compute moments associated with the PDF for λ. However, we consider the quantiles to correspond more closely to the form in which an expert normally expresses his confidence in an evaluation.

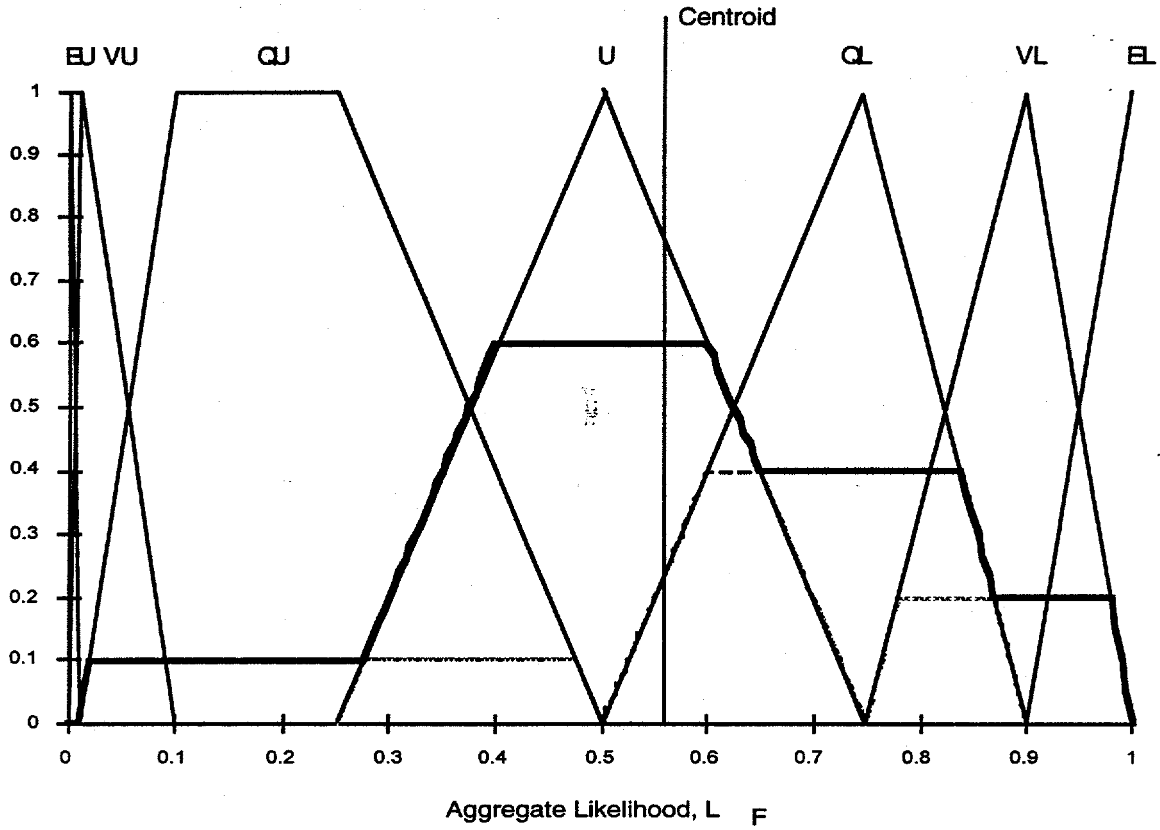


Fig. 3-26. Example of defuzzification of L_F using the centroid method.

3.7.3. Natural Language Expression for Evaluation Result. The result of each MC trial, L_{Fi}, is an estimate for the state vector describing the gas retention status of a tank. The measures $q_i(L_F)$ and $\lambda_{LF}(q)$ are two distinct ways to quantify the aggregation of these trials and to determine quantiles for L_F. However, it is still necessary to specify how these measures are to be used to express the evaluation output.

As noted earlier, we require that the result from the AR model be expressed using a natural language expression. Formally, the natural language expressions of the aggregation measures are denoted as $S(q_i(L_F))$ and $S(\lambda_{LF}(qi))$. where S( ) represents the conversion of the measure to a linguistic parameter. In the case of q_i, the numerical quantity is directly associated with the set that has the highest DOM at this quantile. The name for this set is clearly the natural language expression to use for L_F. For example, suppose that after N trials, the 0.9 quantile vector is

$$q^*_{90} = [0,0,.2,.7,.1,0,0] \ .$$

Then $q_{90}(L_F) = 0.7$ – the DOM associated with {Unresolved}, and the natural language expression associated with this quantile is:

The likelihood of a significant quantity of retained gas at the 0.9 quantile is unresolved.

More precisely, given the PDFs for the primary inputs and the particular AR model used, the probability is 0.9 that the likelihood of a significant quantity of retained gas is unlikely or unresolved.

The centroid is calculated using the membership functions shown in Fig. 3-24, and it is natural to use these functions to obtain $S((\lambda_{LF}(q_i))$. The logical natural language expression for L_F based on the centroid is the set in which it has the maximum DOM. For example, if $\lambda_{L\Delta h}(0.9) = 0.55$, then the likelihood set with the greatest DOM is {Unresolved} and, as before, the result of the evaluation would be expressed as:

The likelihood of a significant quantity of retained gas at the 0.9 quantile is unresolved.

Although $S(q_i(L_F))$ and $S(\lambda_{LF}(q))$ are expressed linguistically, neither one is considered to be fuzzy.

Both approaches to calculating statistics for L_F and for obtaining the associated natural language expressions are useful. The quantile vector approach provides valuable information on how often the various rules in the rule bases "fire" as the quantile changes. One disadvantage associated with this approach is that the dispersion associated with $PDF(L_F)$ is not well-represented. For example, consider the vector

$$q^*90 = [0,0,.95,.95,.96,0,0] \; .$$

In this case, the operations above would lead to a "quite likely" result when an evaluation of "unresolved" would be appropriate. Because of the tendency for multiple elements in the quantile vector to approach 1.0 at high quantiles, this problem is commonplace. The centroid method loses information on the relative growth of membership in the likelihood sets but provides an easily understood measure for estimating statistics and can be transformed back into a natural language expression in a straightforward manner. In Sec. 4 we discuss statistics calculated using both techniques.

The final step in the evaluation process is to compare the natural language expression with some criterion that classifies the tank. That is, we infer the classification of the tank based on the result from the inductive logic structure. This step is an example of a very simple decision model and is the same whether the quantile vector or centroid measure is used. Only the centroid measure will be discussed here. Logically, if $S(\lambda_{LF}(q_i)) \rightarrow$ "quite," "very," or "extremely likely," then the conclusion of the AR model is that the tank fails the screening process. This is consistent with the design of the logic structure and the definition of the output form discussed in Sec. 3.1. Similarly for the "unlikely" expressions, we conclude that the tank passes and that if $S(\lambda_{LF}(q_i)) \rightarrow$ "unresolved," then the tank requires further study. These statements are simple implications with $S(\lambda_{LF}(q_i))$ as the antecedent and can be summarized as follows.

If $S(\lambda_{LF}(q_i))$ is "Extremely Unlikely," "Very Unlikely," or "Quite Unlikely," then the tank passes the screening at the q_i quantile.

If $S(\lambda_{LF}(q_i))$.is "Extremely Likely," "Very Likely," or "Quite Likely," then the tank fails the screening at the q_i quantile.

If $S(\lambda_{LF}(q_i))$ is "Unresolved," then there is insufficient information to classify the tank at the q_i quantile.

In practice one might prefer to use different rules for classification based on degree of conservatism considerations. However, note that the classification decision rules are independent from the evaluation logic structure.

3.8. Summary

In this section, we presented the complete AR model used to illustrate the application of the method to FGWL screening. The starting point is a specification of the scope of the evaluation and the form in which the evaluation is to be expressed. We limited ourselves to an evaluation of gas retention and specified the output, L_F, as the likelihood of a significant quantity of retained gas. The logic structure reflects many of the considerations in the current screening methodology with respect to retained gas.

However, there are significant differences as well, and their influence on example screening results will be discussed later. A universe of discourse, used to represent the element of evidence as a linguistic variable, has been defined for each input to the logic structure. Fuzzy set membership functions were specified to allow conversion of numerical data to the corresponding sets and the technique used to specify set membership for qualitative inputs was described. The relationship between pairs of input parameters and the inferences to be drawn from them is defined explicitly using an implication rule base. The implications are forward-chaining, and at each branch, we specify a consequent, its own universe of discourse, and the specific implications that relate it to its antecedents. This process occurs in parallel for each of the three major logic modules used to evaluate the likelihood of a significant quantity of retained gas based on gas volume predictors, waste characteristic enablers, and GRE indicators. These modules yield the three direct antecedents used to infer the aggregate retained gas likelihood, L_F. Monte Carlo simulation is used to estimate the statistical properties of measures for L_F. These measures can be translated into natural language expressions that are the antecedents in a simple rule base used to classify the tank. A tank can pass or fail the screen or there may be insufficient information to allow a definitive evaluation so the tank status is unresolved. The mechanics of implementing this pilot AR model are discussed in the next section.

4.0. IMPLEMENTATION OF THE APPROXIMATE-REASONING MODEL

The screening model described in Sec. 3 is implemented as a computer program written in the C programming language. The fuzzy rules are evaluated using a modified version of the commercial software package FuzzyCLIPS by Togai InfraLogic, Inc. An overview of the program is given in Fig. 4-1; the basic structure of the computer implementation is as follows.

- Read in data describing the inputs in the algorithm
- Read in the fuzzy rule bases
- For each trial in the Monte Carlo simulation:
  - Select each input from the appropriate distribution
  - Propagate the membership values through the logic structure using the implication rule bases
  - Defuzzify the membership values for the aggregate FGWL likelihood, L_F
  - Place L_F in the appropriate bin
  - Write all values selected from distributions, intermediate membership values, and crisp value of L_F to a file
- Create and store the PDF and cumulative probability distribution function (CDF) from the stored values of L_F
- Post-process with a Microsoft Excel spreadsheet to generate percentile statistics and plots of the PDF and CDF

These aspects of the program are described below.

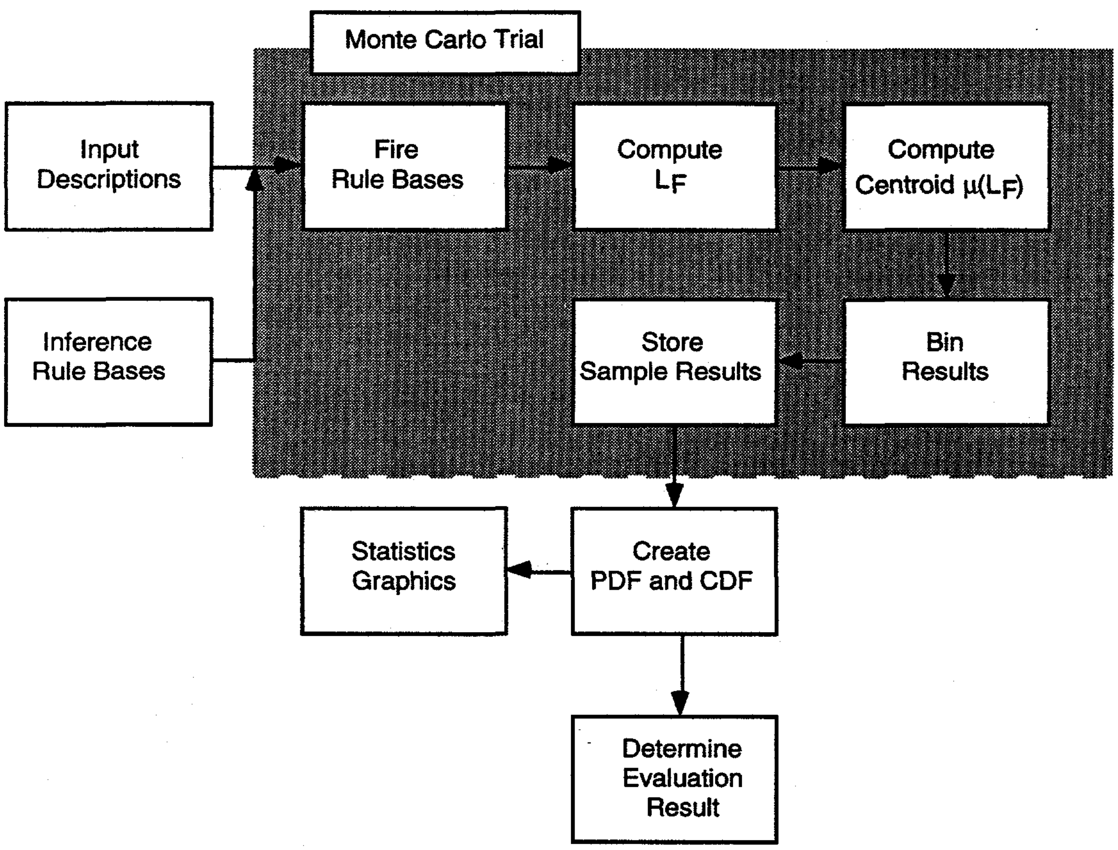


Fig. 4-1. Overall structure for the AR model computer program.

4.1. Reading Input Data

The inputs to the inductive logic structure were discussed in the previous section. Most of these inputs are supplied directly. An example of a directly supplied input is the waste temperature, T. It is described by a Gaussian distribution with a given mean, standard deviation, and upper and lower truncation limits. Other inputs to the logic structure are calculated from intermediate values. For example, the volume of the solids, V_S, is calculated from the salt-cake volume, V_C, and the sludge volume, V_D: $V_S = V_C + V_D$; V_C, and V_D is described by two different distributions.

Table 4-1 lists the basic parameters used in the screening algorithm and shows the direct inputs to the logic structure that they affect. It also lists the type of input, (either a point value or a PDF), which rule the input is used in, the associated figure in Sec. 3 describing the membership functions for that input, and any equations that are used to calculate a primary input to the logic structure.

Table 4-1
Summary of Input Specifications for AR Screening Model

Input to Algorithm	Type of Input	Inputs to Logic Structure	Rule Base (Table)	Membership Function Definition (Figure)	Equations
P_{Si}	Point	P_{Si}	3-1	3-5	3-2
R^2_i	PDF	R^2_i	3-1	3-5	–
S_i	PDF	S_i	3-2	3-5	3-1
$I_{f,m,n}$ I_e	Point Point	$_{f,m,n}$ I_e	3-3 3-3	3-5 3-5	3-2 3-2
h'_{mean}	Point	Δh M_{81} M_E	3-11 3-10 3-10	3-5 3-5 3-5	3-3 3-4 3-5
h_{81mean}	Point	Δh M_{81} M_E	3-11 3-10 3-10	3-5 3-5 3-5	3-3 3-4 3-5
Δh_{81}	PDF	Δh M_{81}	3-11 3-10	3-5 3-5	3-3 3-4
Δh_E	PDF	Δh M_E	3-11 3-10	3-5 3-5	3-3 3-5
V_{Tmean}	Point	Δh M_{81} M_E	3-11 3-10 3-10	3-5 3-5 3-5	3-3 3-4 3-5
V_N	PDF	F_T F_I V_N	3-15 3-18 3-19	3-11 3-13	3-6 3-9
V_S	PDF	V_S	3-18	3-13	–
V_C	PDF	F_T F_I Φ C_M	3-15 3-18 3-18 3-34	3-11 3-13 3-13 3-23	3-6 3-9 3-8 3-10
V_D	PDF	F_T F_I Φ C_M	3-15 3-18 3-18 3-34	3-11 3-13 3-13 3-23	3-6 3-9 3-8 3-10

Table 4-1 (cont)

Input to Algorithm	Type of Input	Inputs to Logic Structure	Rule Base (Table)	Membership Function Definition (Figure)	Equations
Φ_C	PDF	F_T F_I Φ	3-15 3-18 3-18	3-11 3-13 3-13	3-6 3-9 3-8
Φ_D	PDF	F_T F_I Φ	3-15 3-18 3-18	3-11 3-13 3-13	3-6 3-9 3-8
F_{SW}	Point	F_{SCW}	–	–	3-7
q_T	Point	q'''	3-15	3-11	–
T	PDF	T	3-14	3-11	
S	PDF	S	3-15	3-26	–
C_O	PDF	C_O	3-14 3-20	3-11	–
C_g	PDF	C_g	3-25	3-18	–
X_g	Point	X_g	3-25	–	–
N_g	PDF	N_g	3-26	3-18	–
O	PDF	O	3-25	3-18	–
X_O	Point	X_O	3-25	–	–
N_O	PDF	N_O	3-26	3-18	–
δh	PDF	δh	3-29	3-22	–
$N_{\delta h}$	Point	$N_{\delta h}$	3-29	–	–
$\delta\theta$	Point	$\delta\theta$	3-30	–	–
$N_{\delta\theta}$	Point	$N_{\delta\theta}$	3-30	–	–
χ	Point	χ	3-32	3-22	–
C_{Mmean}	Point	C_M	3-34	3-23	3-10

4.2. Reading the Fuzzy Rule Bases

Each of the rule bases described in Sec. 3 is stored in a separate file that is loaded by the program for processing by the AR algorithm. The data format used in describing a rule base is shown in Fig. 4-2. This is the input file for the rule base with the consequent implied by P_S; R^2, P_{RP}, and S as the antecedents; and the likelihood L_{PRS} as the consequent. This logic branch is discussed in Sec. 3.3.1. A brief explanation of the syntax follows.

The first line in Fig. 4-2, [`df rules2 (PRP S) (LPRS)`], defines the name of the rule base [`rules2`] and the names of the inputs [`PRP S`] and outputs [`LPRS`] of the rule base. The next sections [`defUD`] defines the universe of discourse for each input and output. Within this definition are the specifications for each membership function [`dm`] belonging to that universe of discourse. The notation for the set descriptors is the same as in Sec. 3. All the membership functions in this rule base are made up of [`linear`] segments. For example, the definition of the fuzzy set Very Negative (VN) of the slope variable, S, is given by the line `(dm VN (linear -2.0 1.0 -1.0 1.0 -0.25 0.0) )`. The values following `linear` are pairs of points describing the shape of the membership function. That is, the set VN starts at a value of -2.0 with membership 1.0, then goes to -1.0 with membership 1.0, and stops at -0.25 with membership 0.0 (see Fig. 3-5). After all of the universes of discourse are defined, the rules are defined [`dfr`]. The rule `(dfr r1 (PRP is PRP-UD::H) (S is S-UD::P) => (LPRS is LPRS-UD::U) )` translates as:

If P_{RP} is High (H) and S is Positive (P), then L_{PRS} is Unresolved (U) (see Table 3-4).

```
(df rules2 (PRP S) (LPRS)

(defUD PRP-UD (-1.0 1.0)
(dm L (linear -1.0 1.0 0.05 1.0 0.3 0.0) )
(dm U (linear 0.05 0.0 0.3 1.0 0.5 1.0 0.7 0.0) )
(dm H (linear 0.5 0.0 0.7 1.0 1.0 1.0) )
)

(defUD S-UD (-2.0 1.0)
(dm VN (linear -2.0 1.0 -1.0 1.0 -0.25 0.0) )
(dm SN (linear -1.0 0.0 -0.25 1.0 0.0 1.0 0.25 0.0) )
(dm P  (linear 0.0 0.0 0.25 1.0 1.0 1.0) )
)

(defUD LPRS-UD (0.0 1.0)
(dm VU (linear 0.0 1.0 0.01 1.0 0.1 0.0) )
(dm QU (linear 0.01 0.0 0.1 1.0 0.25 1.0 0.5 0.0) )
(dm U  (linear 0.25 0.0 0.5 1.0 0.75 0.0) )
(dm QL (linear 0.5 0.0 0.75 1.0 0.9 0.0) )
(dm VL (linear 0.75 0.0 0.9 1.0 1.0 1.0) )
)

(dfr r1 (PRP is PRP-UD::H) (S is S-UD::P)  => (LPRS is LPRS-UD::U)  )
(dfr r2 (PRP is PRP-UD::H) (S is S-UD::SN) => (LPRS is LPRS-UD::QL) )
(dfr r3 (PRP is PRP-UD::H) (S is S-UD::VN) => (LPRS is LPRS-UD::VL) )
(dfr r4 (PRP is PRP-UD::U) (S is S-UD::P)  => (LPRS is LPRS-UD::QU) )
(dfr r5 (PRP is PRP-UD::U) (S is S-UD::SN) => (LPRS is LPRS-UD::U)  )
(dfr r6 (PRP is PRP-UD::U) (S is S-UD::VN) => (LPRS is LPRS-UD::QL) )
(dfr r7 (PRP is PRP-UD::L) (S is S-UD::P)  => (LPRS is LPRS-UD::VU) )
(dfr r8 (PRP is PRP-UD::L) (S is S-UD::SN) => (LPRS is LPRS-UD::QU) )
(dfr r9 (PRP is PRP-UD::L) (S is S-UD::VN) => (LPRS is LPRS-UD::U)  )

)
```

Fig. 4-2. Rule base for combining P_{RP} and S to generate L_{PRS}.

Defining all the rules completes the definition of the rule base. Note that in this case, P_{RP} and L_{PRS} are intermediate variables and the membership functions defined here are merely placeholders. They are not used in the actual computations as defined here but, as explained below, are replaced dynamically.

4.3. Monte Carlo Simulation

At the start of every MC trial, each input parameter is selected randomly from the appropriate distribution, and the primary inputs to the logic structure are calculated. For parameters described by a normal distribution, values are chosen using a routine from Press (1992). Values are chosen from uniform distributions using the "random" routine in the standard C library.

The rule bases are evaluated in succession. For the first layer in the tree, the inputs are supplied directly, and the result is a set of membership values for the output of these rules. For example, the values for the total organic concentration, C_O, and the waste temperature, T, are expressed as DOMs in their respective universes of discourse and are input to the rule base represented in Table 3-14 to infer the thermolysis gas generation potential, G_T. The output from the evaluation of the rule base is a set of membership values (High, Medium, Low) for G_T. For successive sets of rules, these membership values are passed on directly to the next rule as the input. For example, the rule represented in Table 3-16 for combining G_R and G_T to generate the aggregate generation potential, G, is evaluated using the membership values for G_R and G_T. Because of this method of passing on membership values, actual membership functions are not needed for intermediate variables in the fuzzy rule tree.

Modifications to the FuzzyCLIPS source code were required to implement this method of passing membership function from one rule base to another. FuzzyCLIPS has a routine that accepts a rule base and DOM values for the inputs and returns a point output value. This is the process of defuzzification described in Sec. 3.7.2, and the point output is the centroid. This routine was modified to return the membership values of the fuzzy sets representing the output instead.

Another required modification deals with the intermediate rule bases. For these rule bases, the input is not described by a point value but by a vector of membership values. Because the routine that evaluates the rule base requires a point value for the input, dummy membership functions are created. These dummy membership functions are constructed so that a point value of 0 produces the correct membership in each set for that input. For example, to pass on the membership values of (Low = 0.4, Unresolved = 0.5, High = 0.1) for P_{RP}, the following dummy membership functions are created.

```
(dm L (linear -1.0 0.0 0.0 0.4 1.0 1.0) )
(dm U (linear -1.0 0.0 0.0 0.5 1.0 1.0) )
(dm H (linear -1.0 0.0 0.0 0.1 1.0 1.0) )
```

These membership functions replace the ones loaded from the file described in Fig. 4-2. This is done by excising the old membership functions from the rule base definition and loading the new definitions from a file. Because of the way membership functions are linked to rules internally in FuzzyCLIPS, the rules also need to be excised and reloaded.

As discussed in Sec. 3.7, it is necessary to compute measures for L_F to obtain the statistics for the Monte Carlo simulation. Calculation of the centroid measure, λ_{LF}, requires the use of the membership functions shown in Fig. 3-25. After the centroid is calculated, it is binned (200 bins covering the range 0 to 1), which completes one MC trial. A total of 2000 trials was run. For each trial, the values for each input variable to the algorithm, the membership values for all the intermediate variables, and the crisp value of L_F are written to a text file. This file is later processed with Microsoft Excel to generate the statistics used in Sec. 5.

4.4. Creating and Storing the PDF and CDF of L_F

The binned values of L_F are used to estimate the PDF and a CDF for L_F. These values then are written to a text file that can be read by Microsoft Excel. Plots of the PDF and CDF of L_F are generated using Excel.

4.5. Implementation Issues

The AR program was run on an IBM PC 486 66-MHz computer with 16 MB of RAM to generate the results discussed in Sec. 5. Running the entire algorithm for a tank requires about 6 h of computing time for 2000 MC trials. Running just the barometric pressure logic submodule requires between 30 and 80 min per tank, depending on the number of level sensors (one to four) that were available for each tank.*

It should be noted that the entire fuzzy rule tree is exactly equivalent to a single rule base with 40 inputs and one output (L_F) but containing $3^{38} \bullet 4 \bullet 5$ ($2.7 \bullet 10^{19}$) rules.** Although it is impossible to actually construct this huge rule base, it is possible to create smaller rule bases that combine three or four inputs. For example, the rules applying to one instrument in the L_B module, Fig. 3-4, can be combined into one

*More recently, some simulations were rerun on a PentiumPro 200-Mhz PC with 64 MB of RAM. The time required to run the entire algorithm was reduced to a little over 2 h. The run time for the barometric pressure module was reduced to between 8 and 22 min.

** One input is described by five membership functions; one is described by four membership functions; and the other 38 are described by three membership functions.

rule base with four inputs (P_S, R^2, S, I), one output (L_i), and 81 rules. Using these condensed rule bases to run the barometric pressure logic submodule resulted in about a 10% to 30% reduction in computing time. This is primarily a result of reducing the amount of I/O associated with rewriting and reloading the membership functions and rules for the intermediate variables. No significant reduction in computing time was observed for the full algorithm. The benefits of reduced I/O were offset by the increased time required to evaluate rule bases with up to 81 rules.

4.6. Equations Used in the Program

The equations used in the simulation are given below. normal(low, high, mean, sd) is a routine that returns a random value from a normal distribution described by mean and sd and truncated at low and high. uniform(low, high) is a routine that returns a random value from a uniform distribution between low and high.

$A = V_{Tmean} / h'_{mean}$
$V_{81} = h_{81mean} * A$
$V_{81low} = V_{81} - V_{Tr} * V_{Tsd}$
$V_{81high} = V_{81} + V_{Tr} * V_{Tsd}$
$\alpha = C_{Mmean} / (V_{Dmean} + V_{Cmean})$

h_{81} =normal(V_{81low}, V_{81high}, V_{81}, V_{Tsd}) / A
Δh_{81} =normal(Δh_{81low}, Δh_{81high}, Δh_{81mean}, Δh_{81sd})
Δh_E =normal(Δh_{Elow}, Δh_{Ehigh}, Δh_{Emean}, Δh_{Esd})
V_N =normal(V_{Nlow}, V_{Nhigh}, V_{Nmean}, V_{Nsd})
V_C =normal(V_{Clow}, V_{Chigh}, V_{Cmean}, V_{Csd})
V_D =normal(V_{Dlow}, V_{Dhigh}, V_{Dmean}, V_{Dsd})
Φ_C =normal(Φ_{Clow}, Φ_{Chigh}, Φ_{Cmean}, Φ_{Csd})
Φ_D =normal(Φ_{Dlow}, Φ_{Dhigh}, Φ_{Dmean}, Φ_{Dsd})
T =normal(T_{low}, T_{high}, T_{mean}, T_{sd})
S =normal(S_{low}, S_{high}, S_{mean}, S_{sd})
C_O =normal(C_{Olow}, C_{Ohigh}, C_{Omean}, C_{Osd})
C_g =normal(0.0, ∞, C_{gmean}, C_{gsd})
N_g = uniform(N_{glow}, N_{ghigh})
O =normal(O_{low}, O_{high}, O_{mean}, O_{sd})
N_O = uniform(N_{Olow}, N_{Ohigh})
δh =normal(δh_{low}, δh_{high}, δh_{mean}, δh_{sd})
δh = uniform(δh_{low}, δh_{high})

$V_T = V_N + V_C + V_D$

$h' = V_T / A$
$\Delta h_M = h' - h_{81}$
$\Delta h = \Delta h_M + \Delta h_{81} + \Delta h_E$
$M_{81} = | \Delta h_{81} / \Delta h_M |$
$M_E = | \Delta h_E / \Delta h_M |$

$q''' = q_T / V_T * 1000$
$V_I = \Phi_D * V_D + \Phi_C * V_C * f_{SCW}$
$F_T = (V_N + V_I) / V_T$
$F_S = V_I / (V_T - V_N)$
$\Phi = (\Phi_D * V_D + \Phi_C * V_C) / (V_D + V_C)$

$C_M = \alpha * (V_D + V_C)$

5.0. RESULTS OF THE APPROXIMATE-REASONING MODEL ALGORITHM TESTING

To demonstrate the utility of the AR approach to FGWL screening, we considered two problems.

1. A complete tank evaluation in which the entire algorithm is used. This was done for two tanks, U-106 and AW-104. U-106 is a single-shell tank with large sludge and saltcake layers. AW-104 is a double-shell tank with over 1 million gallons of supernate. Both of these tanks had been recommended for inclusion on the FGWL as a result of the Hodgson (1995) screening method as applied by Barton (1996).
2. Partial evaluations using the barometric pressure correlation logic submodule for all of the tanks on the FGWL and those flagged previously by Whitney but not currently on the FGWL.

Together, the results from these two problems illustrate the power of an AR model for screening and provide a meaningful comparison of the approach used here with existing methods.

5.1. Results from Complete Tank Evaluations

In a complete evaluation, all of the major logic branches described in Sec. 3.2 are used. This requires that the 40 parameters needed to calculate the primary inputs to the logic tree be specified. Both quantitative and qualitative factors are input. Quantitative factors are used in the evaluation of the predictor and enabler likelihoods, L_P and L_E. Qualitative factors are used to determine the indicator likelihood, L_I. For testing purposes, these qualitative parameters were specified so that there would be no clear evidence of GRE behavior. This is consistent with the data available for Tanks U-106 and AW-104. Many of the quantitative primary inputs and several of the qualitative inputs are represented as random variables. The approach to representing these variables in the model is described in Sec. 3.7 and in Sec. 4.

The results from a single MC trial for Tank U-106 are discussed in Appendix C. This discussion explains in detail how the final aggregate likelihood is arrived at for a particular set of input parameters. Readers interested in understanding what specific inferences are drawn from each implication rule base in the model should read this appendix.

As noted above, for each MC trial, values for all of the primary inputs are obtained using random sampling from the defining PDFs. A complete evaluation is carried out using these values. This constitutes one trial; 2000 trials were used in the simulations for Tanks U-106 and AW-104. In this section, we are concerned primarily with a discussion of the statistics associated with the Monte Carlo simulations, the natural language expressions for the statistical measures, and the final screening classification for the tanks. Recall from Sec. 3.7 that two approaches are used to calculate statistics for L_F. In the first, we estimate PDFs for the DOMs in each set used to describe L_F,

$$PDF(L_F) = [PDF(\gamma(S_j)\ j = 1,7)] \ , \tag{5-1}$$

and use the vector

$$q_i^* = [q_i(\gamma(S_j)\ j = 1,7)] \ . \tag{5-2}$$

as a representation for the ith quantile of L_F. In the second approach, we use the centroid value, λ_{LF} and estimate a PDF for it as well from the simulation. The ith quantile for the centroid is $\lambda_{LF}(q_i)$. Means, standard deviations, and the CDFs also are computed. We will refer to these two approaches to calculating statistics as the vector measure and centroid measure.

5.1.1. Evaluation for Tank U-106. Statistics for the aggregate likelihood, L_F, computed for both the membership vector and centroid measures are given in Table 5-1 for Tank U-106. These include the 0.25, median, 0.75 and 0.95 quantiles; the mean; and the standard deviation. Also shown are the corresponding

Table 5-1
Statistics for L_F Generated in the Evaluation of U-106 from a Monte Carlo Simulation with 2000 Trials

Statistic	Membership Vector Measure			Centroid Measure		
	Degrees of Membership (EU,VU,QU, U,QL,VL,EL)	Natural Language Expression	Screening Result	Centroid	Natural Language Expression	Screening Result
Median, $q_{0.5}$	{0,0,0,.07,.5,.25,0}	Quite Likely	Fail	.72	Quite Likely	Fails
$q_{0.25}$	{0,0,0,0,.43,.16,0}	Quite Likely	Fail	.62	Unresolved	Insufficient Evidence
$q_{0.75}$	{0,0,0,.44,.53,.29,0}	Quite Likely	Fail	.74	Quite Likely	Fails
$q_{0.95}$	{0,0,0,.55,.56,.5,0}	Quite Likely	Fail	.76	Quite Likely	Fails
Mean	{0,0,0,.19,.44,. 24,0}	Quite Likely	Fail	.68	Quite Likely	Fails
Standard Deviation[1]	{0,0,0,.22,.16,.13,0}	—	—	.08*	—	—

natural language expressions for the result at each quantile and the mean and the screening result obtained using the criteria given in Sec. 3.7.3. The evaluation result for this tank is quite clear—Tank U-106 fails the pilot AR model screen at the median and for larger quantiles. Note also that for q_{95}, the DOMs are almost the same for {Unresolved}, {Quite Likely}, and {Very Likely}. This tendency for more than one set to have relatively high membership is the main reason for calculating the centroid statistics.

Figure 5-1 shows the CDF for $\lambda(L_F)$. Note that between the median and the 0.95 quantile, q_{95} the value of $\lambda(L_F)$ barely increases from 0.72 to 0.76. This indicates that for this tank, the evaluation result is not very sensitive to the upper tails of the input PDFs. It should be noted that any such conclusions are subject to the validity of the logic rules as well as the PDFs and membership functions used for the inputs to the logic structure. Here all of these classes of objects as well as the logic structure itself are intended for testing purposes only.

Insight into why this result is inferred from the evidence can be seen by examining the degree of membership vectors from the primary logic modules.* This can be done at any quantile. We use the median statistics, shown in Fig. 5-2, as a reasonable approximation of a best-estimate evaluation (Myers and Booker 1991).** As noted earlier, the indicator input parameters were chosen to provide no positive or negative indications of GRE behavior. Therefore, the aggregate likelihood is inferred only from the predictor and enabler logic modules. In this case, the component likelihoods in the predictor module—the barometric pressure and long-term level change likelihoods—have only large DOMs in {Unresolved}. However, the waste characteristics are such that the potential for gas retention is inferred to be high, and the gas generation potential also is judged to be significant. This results in an enabler likelihood with the largest membership by far in {Quite Likely}. The convolution of these likelihoods leads to membership for L_F in {Quite Likely} of $\gamma = 0.5$ and in {Very Likely} $\gamma = 0.25$.

*The details of the evaluations carried out in the three logic modules are given in Appendix C.
**Readers familiar with the min-max rule will notice that the DOM values are transmitted only approximately across a logic junction. This is because the statistics for each likelihood vector measure are computed separately.

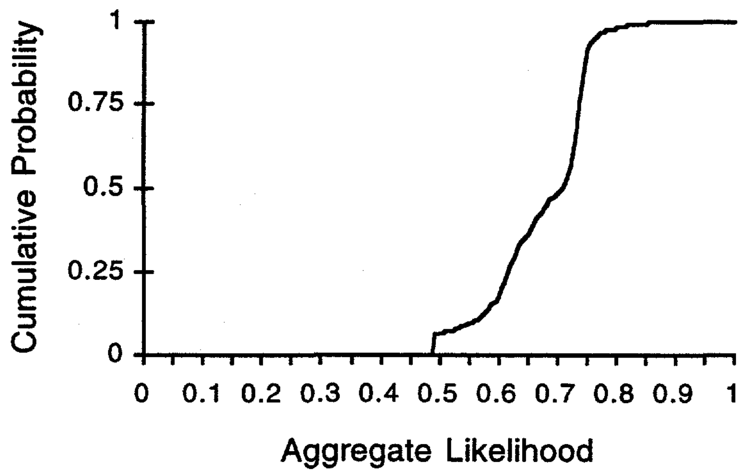


Fig. 5-1. Cumulative distribution function for the centroid measure of L_F obtained for the MC simulation of Tank U-106 with the AR model.

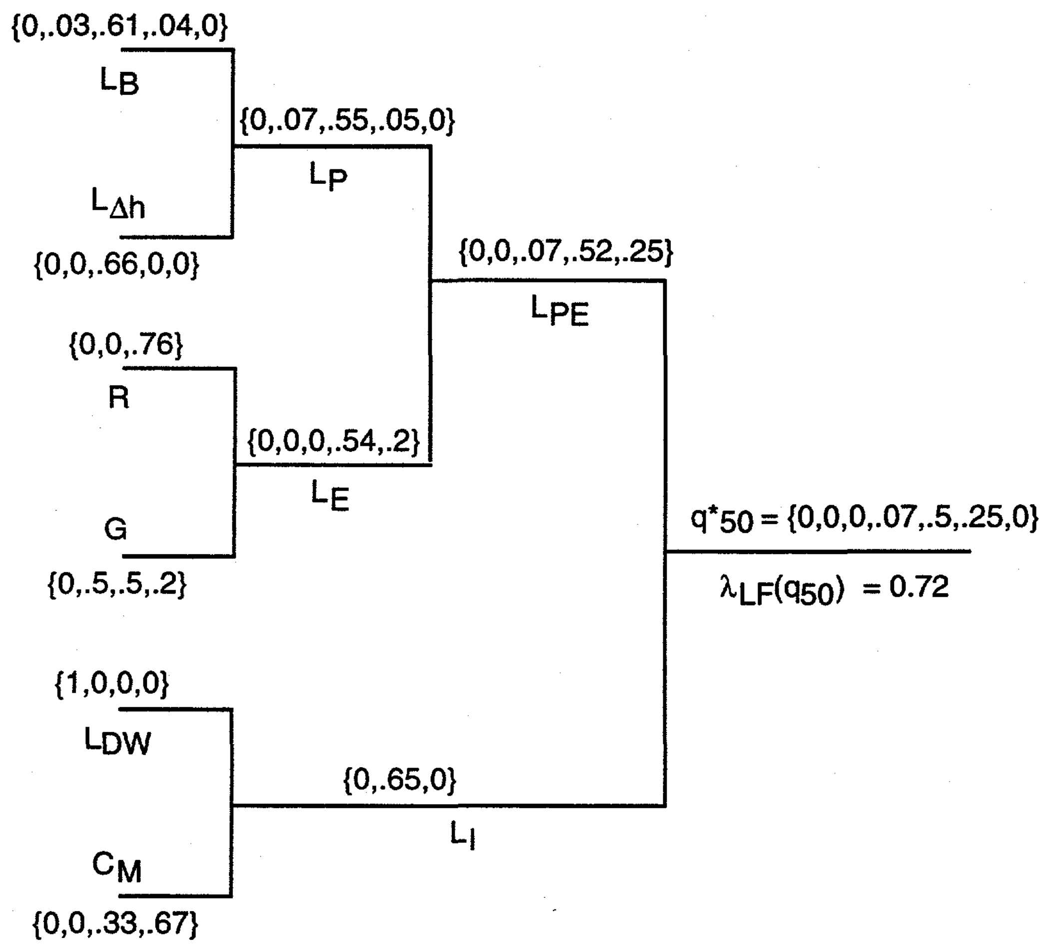


Fig. 5-2. Median statistics computed for Tank U-106.

5.1.2. Evaluation for AW-104. We performed a complete evaluation for Tank AW-104 as well. The statistics for the vector and centroid measures for L_F are given in Table 5-2. For this tank, the centroid measures are $\lambda_{LF}(q_{50}) = 0.29$ and $\lambda_{LF}(q_{95}) = 0.40$. The natural language expression for both results is "quite unlikely," and the tank passes the screening criteria at these confidence levels. The CDF for $\lambda(L_F)$ is shown in Fig. 5-3.

Table 5-2
Statistics for L_F Generated in the Evaluation of Tank AW-104 from a Monte Carlo Simulation with 2000 Trials

	Membership Vector Measure			Centroid Measure		
Statistic	Degrees of Membership	Natural Language Expression	Screening Result	Centroid	Natural Language Expression	Screening Result
Median, $q_{0.5}$	{0,0,.5,.12,0,0,0}	Quite Unlikely	Pass	.29	Quite Unlikely	Pass
$q_{0.25}$	{0,0,.5,.06,0,0,0}	Quite Unlikely	Pass	.26	Quite Unlikely	Pass
$q_{0.75}$	{0,.22,.54,.2,.06,0,0}	Quite Unlikely	Pass	.33	Quite Unlikely	Pass
$q_{0.95}$	{0,.53,.56,.50,.15,0,0}	Quite Unlikely	Pass	.40	Quite Unlikely	Pass
Mean,	{0,.12,.46,.16,.04,0,0}	Quite Unlikely	Pass	.29	Quite Unlikely	Pass
Standard Deviation	{0,.20,.13,.14,.05,.02,0}	—	—	.07	—	—

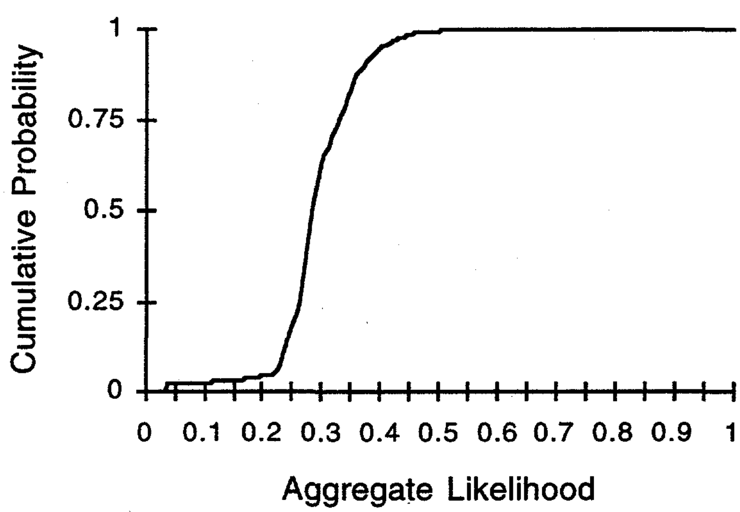


Fig. 5-3. Cumulative distribution function computed for $\lambda(L_F)$ for AW-104.

Figure 5-4 shows the propagation of the DOM vectors for the median statistics. Although both L_P and L_I have non-zero DOMs only in {Unresolved}, the enabler likelihood has a median degree of membership in {Quite Unlikely} of $\gamma(L_E,QU) = 0.55$. According to the logic rules used here, this has a strong influence on the final result. The median vector measure for the aggregate likelihood is q^*_{50} = {0,0,.5,.12,0,0,0}. Note that for both tanks, the classification is shifted from "unresolved" by the enabler module inferences.

In calculating the centroid measure for L_F we used the asymmetric membership functions shown in Fig. 3-25. Recall that these functions were chosen to illustrate how output membership functions can be specified to incorporate degree-of-conservatism considerations. We also have calculated centroid measures using the symmetric membership functions shown in Fig. 5-5. Table 5-3 compares the results using the two different membership function specifications. The use of symmetric functions increases the centroid measure for L_F slightly. The effect is to change the natural language expression for L_F from "quite unlikely" to "unresolved" at the 0.95 quantile. Note that the membership vector measure is unaffected by the change in membership functions for L_F.

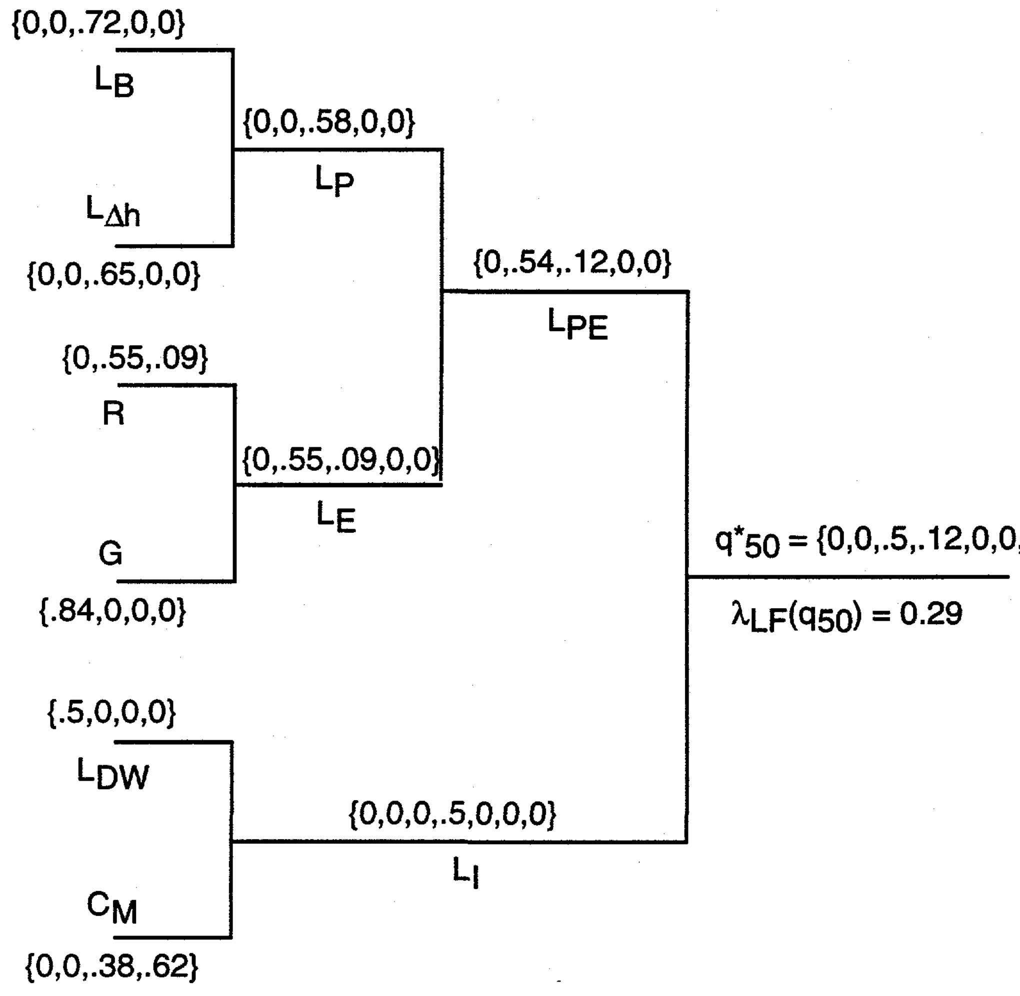


Fig. 5-4. Median statistics computed for Tank AW-104.

Table 5-3
Comparison of Centroid Measures Using Different Membership Functions

	Asymmetric Centroid Measure			Symmetric Centroid Measure		
Statistic	**Centroid**	**Natural Language Expression**	**Screening Result**	**Centroid**	**Natural Language Expression**	**Screening Result**
Median, $q_{0.5}$	.29	Quite Unlikely	Pass	.34	Quite Unlikely	Pass
$q_{0.25}$	.26	Quite Unlikely	Pass	.30	Quite Unlikely	Pass
$q_{0.75}$	.33	Quite Unlikely	Pass	.38	Quite Unlikely	Pass
$q_{0.95}$	.40	Quite Unlikely	Pass	.45	Unresolved	Insuf-ficient Evidence
Mean	.29	Quite Unlikely	Pass	.34	Quite Unlikely	Pass
Standard Deviation	.07	—	—	.07	—	—

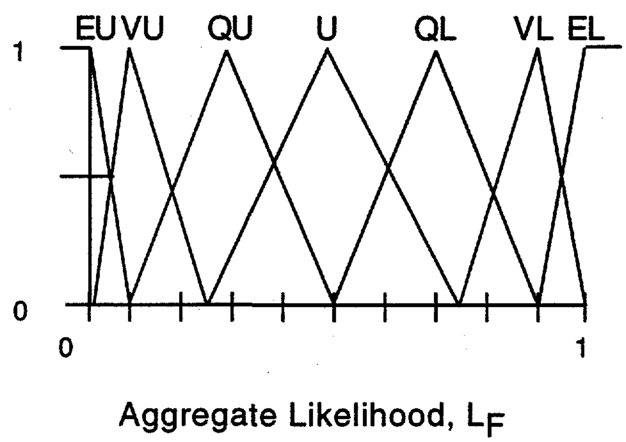


Fig. 5-5. Symmetric membership functions for L_F.

5.1.3. Discussion of Full Tank Evaluations. It is interesting to compare these results for Tanks U-106 and AW-104 with Hodgson's evaluation (summarized in Table 5-4). Hodgson selected 25% of LFL as his screening threshold and uses the long-term surface-level rise and the barometric pressure slope to obtain two separate estimates for the gas concentration in the dome space (see Sec. 2.2). In the case of the barometric pressure correlation-based estimate, the level sensor that gives the highest gas concentration is used.* Tank AW-104 fails both tests and Tank U-106 fails the surface-level rise screen.

Direct comparisons of the Hodgson results and the test evaluations presented here are problematic. Our approach incorporates most of the primary inputs used by Hodgson but evaluates them quite differently. However, several points can be made based on a very rough parallel between the gas concentration estimates and specific component likelihoods.

*If only one sensor fails the Whitney flag, then the gas concentration based on that sensor is used.

Table 5-4
Screening Results from Hodgson

Tank	Judgment	Surface Level Rise Model (% LFL)	Barometric Pressure Model (% LFL)	Sensor used for BP Model
AW-104	Fail	127	134	FIC
U-106	Fail	37	20	FIC

Both tanks fail the Hodgson screen based on level rise, which we denote as $LFL_{\Delta h}$. In our analysis of Tank AW-104, the vector measure at the median for $L_{\Delta h}$ is $q^*_{50} = \{0, 0, .65, 0, 0\}$.† That is, there is only membership in the unresolved fuzzy set. For this tank, the effect of the correction for evaporation is large, with $\gamma(M_E, \text{High}) = 1$. The logic rule for the quality Q always evaluates to "poor" in this case; therefore, $L_{\Delta h}$ can have membership only in {Unresolved} regardless of the numerical value for Δh (see Table 3-11). The same result occurs for Tank U-106 except that the correction for the pre-1981 level is the determining factor. Thus, it can be seen that the pilot AR model judges the long-term level data for these tanks to be inadequate to allow any strong inference to be made.

The approach taken in the current work with the barometric pressure-level correlation can be viewed as an extension of that in Whitney (1995). In addition to the fraction of negative slopes probability, P_S, we use two other measures, the regression coefficient, R^2, and the level-pressure slope, S, calculated by Whitney. An additional explicit judgment is made about the quality of the inference to be drawn from this evidence by also considering the number of intervals from which these statistics were generated.* Further, the barometric pressure logic submodule incorporates the evidence from all available level sensors into an aggregate likelihood.

In Hodgson, the volume of gas is calculated using a simple linear model for pressure and height. The volume used in screening is based on the instrument with the largest negative slope that failed the Whitney flag, $P_S < 0.05$. Thus, although S is used in this approach, instrument quality is not considered, and a mechanism for reconciling conflicting data is not implemented.

For Tank AW-104, Whitney calculates P_S= 0.0 and 0.34 for the FIC (f) and manual tape (m), respectively. There is no ENRAF (e) or neutron log (n) data. This means that the tank fails the Whitney flag based on the FIC, and a gas concentration based on a slope of S = -0.20 is calculated.** The flammable gas concentration calculated using this slope is above the threshold defined in Hodgson, so the tank fails the screen. In the evaluation performed with the AR model, the median vector measures based on the FIC and MT are $q^*_{50}(L_f) = \{0, 0, .88,0,0\}$ and $q^*_{50}(L_m) = \{0, .44, 0, 0, 0\}$, respectively. The primary consideration here is L_f. Although P_S is very low, the evaluation algorithm also considers the fact that the linear regression coefficient is small, as is the slope. According to the AR rule bases, the likelihood to be inferred from this set of values should have only a non-zero DOM in {Unresolved}. This is indeed the case, $q^*_{50}(L_B) = \{0,0,.71,0,0\}$. We can consider the information available from the MT as of insufficient strength to change this evaluation.

Tank U-106 passes the Whitney flag using the ENRAF data but fails when either the FIC or NL data are used. The gas concentration is calculated using the FIC with a slope of S = -0.15.*** Figure 5-6 shows

† The degree of conservatism in the estimate for $LFL_{\Delta h}$ is difficult to estimate, and we arbitrarily chose to compare it with the median likelihood estimate.

*Note that although P_S includes the number of intervals directly, the other statistics do not. Also, although the probability distributions for R^2 and S provide information about the variability of these statistics, they provide no direct information on how many data points were used in their construction.

**This slope value represents the 0.75 quantile. The reason for using this quantile is not clear.

***It is unclear why the slope for the neutron log was not used as the absolute magnitude of the slope for this instrument is greater.

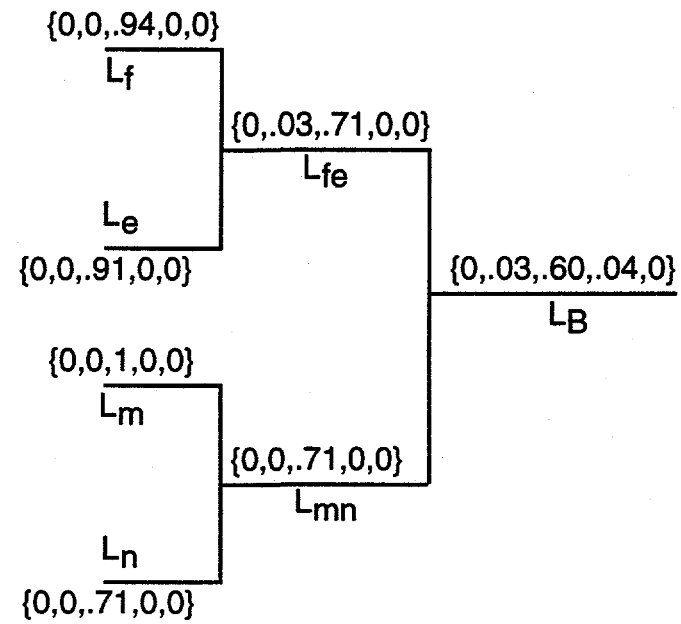


Fig. 5-6. Median vector statistics for barometric pressure logic submodule for Tank U-106.

the median vector measures calculated for the barometric pressure submodule. Only non-zero membership in the unresolved set is seen for all three instruments. In the case of the FIC, the low value of $P_S = 0.002$ is given less weight because both R^2 (0.17) and S (-0.07) are small. For the ENRAF, the small number of intervals drives the evaluation toward {Unresolved}. The neutron log results are particularly interesting. In this case, we have $P_S = .035$, and the median values for R^2 and S are 0.237 and -0.662, respectively. The vector measure of retained gas inferred from these three measures is $q^*_{50}(L_{PRS}) = \{0, 0, .1, .46, 0\}$. This likelihood is relatively high. However, the number of intervals is small (I = 8), so according to the logic incorporated in the AR rule base (Table 3-5), the result must be relaxed toward unresolved. The median vector measure inferred from the individual sensor vectors is $q^*_{50}(L_B) = \{0, .03, .6, .04, 0\}$. Thus, the result obtained here is considerably different from the conclusion reached in Hodgson for this tank based on the barometric pressure data.

5.2. Evaluation of Barometric-Pressure-Based Likelihood Logic Submodule for Selected Tanks

Because of time constraints, complete evaluations could be performed only for two tanks using the AR model described here. This was not considered a serious problem because the rules and membership functions used are intended for testing purposes only. However, the barometric pressure likelihood logic submodule was developed first and consequently is more mature. In addition, its relationship to Whitney's work, as discussed above, is reasonably clear. Therefore, we decided to carry out an evaluation of L_B for the 25 tanks on the FGWL as well as the additional 37 tanks that were flagged by Whitney during his study. Of these 37 tanks, 24 originally were recommended by Barton for inclusion on the FGWL.* We consider these three sets of tanks separately below.

5.2.1. Membership Functions for L_B. In a complete tank evaluation, the only measure available for computing statistics for L_B is the membership vector. To use the centroid measure to evaluate tanks with just the barometric pressure logic submodule, it is necessary to specify membership functions for L_B. These are shown in Fig. 5-7 and correspond to the symmetric membership functions used earlier for L_F in Fig. 5-5 but without the "extremely" hedges.

* Tank BY-106 was not flagged by Whitney but was included on the Barton list.

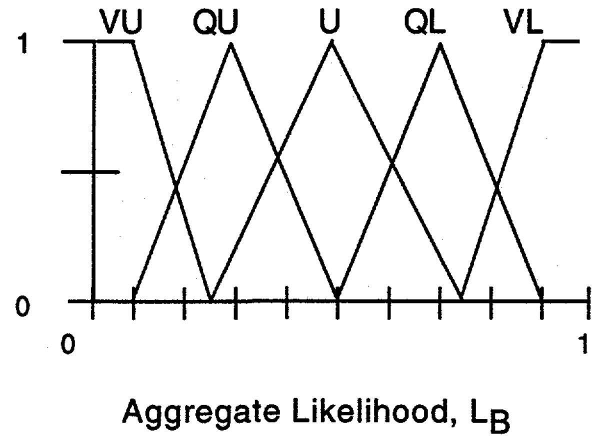


Fig. 5-7. Membership functions for L_B.

The screening criteria to which these expressions are compared are the same as for L_F. For values of $\lambda(L_B)$ that correspond to the natural language expression "unresolved," an additional classification step was used. It was found that this classification could be subdivided based on either the shape of the CDF or the magnitude of the standard deviation. Figure 5-8 shows the two distinctive CDF curves that can occur for tanks that are classified as "unresolved." For the curve labeled UP, corresponding to the expression "unresolved with poor evidence," it can be seen that for small values of cumulative probability P, the value of $\lambda(L_B)$ is 0.5. This is also the value at the median. Logically, this represents the case where the evidence is judged to be of insufficient quality to allow a judgment regardless of the numerical values of the primary inputs. For the curve labeled UG, "unresolved good evidence," the value of $\lambda(L_B)$ depends on the distributions of the primary inputs. In this case, the evaluation is that the evidence is acceptable but allows no definitive inference of the likelihood of a significant quantity of retained gas to be drawn. The shape of the cumulative distribution in the UP case implies a small value for the standard deviation, σ. Therefore, by examining the results, it is possible to classify tanks as being in UP with the simple threshold criterion that a tank is a member of UP if $\sigma \leq 0.025$.

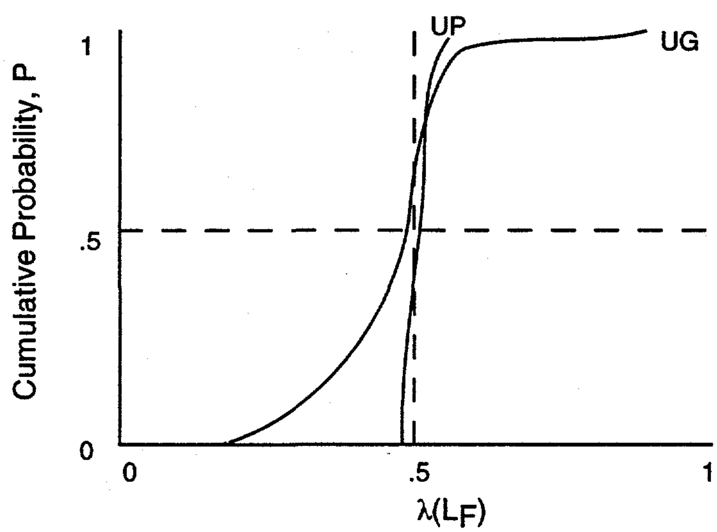


Fig. 5-8. Shapes of cumulative probability distributions for λ_L for tanks classified "unresolved, poor evidence" (UP) and "unresolved, good evidence" (UG).

5.2.2. L_B for Tanks on the FGWL. Table 5-5 lists the tanks on the FGWL. Also shown are the median and 0.25, 0.75, and 0.95 quantiles and the mean and the standard deviation for λ_{LB}. Values of λ_{LB} at the median range from 0.23 to 0.79 and from 0.33 to 1.00 at q_{95}. Also shown is the result of the comparison with the screening criteria for λ_{LF} but applied only to the centroid computed with the barometric pressure logic submodule.

Classification of a tank depends on the quantile of the CDF used. Table 5-6 shows the groupings based on the median and 0.95 quantiles for L_B. The total number of tanks is fixed, and the number of tanks classified as UP is not a function of the quantile considered. Thus, it can be seen in this instance that the effect of increasing the quantile from 0.5 to 0.95 is to move six tanks from UG to F. Twenty-one of the FGWL tanks also were flagged by Whitney. The logic rules and membership functions used in testing the AR algorithm make it somewhat more difficult for a tank to be classified as failing the screening. In the absence of relatively strong evidence for gas retention, a tank will be placed in one of the unresolved sets. Table 5-7 shows the four tanks not flagged by Whitney along with the current evaluation results. For Tanks S-112 and T-110, both methods yield the same result, whereas for Tanks AX-101 and SX-109, the current evaluation is to classify these tanks as unresolved.

Table 5-5
Results for L_B for the Tanks on the FGWL from Monte Carlo Simulations with 2000 Trials

Tank	Centroid Quantile				Mean	Std. Deviation	Classification*	
	0.25	0.50	0.75	0.95			q_{50}	q_{95}
A101	0.500	0.500	0.500	0.513	0.502	0.011	UP	UP
AN103	0.547	0.641	0.755	0.884	0.659	0.131	F	F
AN104	0.466	0.500	0.517	0.606	0.479	0.095	UG	F
AN105	0.500	0.500	0.540	0.673	0.518	0.078	UG	F
AW101	0.500	0.500	0.502	0.568	0.500	0.041	UG	UG
AX101	0.356	0.500	0.500	0.562	0.457	0.076	UG	UG
AX103	0.413	0.500	0.514	0.589	0.466	0.096	UG	UG
S102	0.660	0.786	0.879	0.963	0.767	0.136	F	F
S111	0.688	0.759	0.803	0.840	0.737	0.086	F	F
S112	0.202	0.228	0.235	0.333	0.218	0.076	P	P
SX101	0.500	0.500	0.507	0.563	0.511	0.024	UP	UP
SX102	0.500	0.565	0.609	0.712	0.573	0.066	UG	F
SX103	0.644	0.776	0.897	1.000	0.766	0.154	F	F
SX104	0.528	0.600	0.729	0.892	0.615	0.148	F	F
SX105	0.500	0.532	0.624	0.714	0.568	0.081	UF	F
SX106	0.566	0.684	0.793	0.892	0.681	0.134	F	F
SX109	0.500	0.500	0.500	0.500	0.500	0.000	UP	UP
SY101	0.554	0.610	0.708	0.841	0.631	0.115	F	F
SY103	0.368	0.509	0.560	0.647	0.465	0.144	UG	F
T110	0.274	0.286	0.310	0.405	0.292	0.065	P	P
U103	0.699	0.771	0.825	0.872	0.757	0.085	F	F
U105	0.509	0.590	0.713	0.870	0.610	0.148	UG	F
U107	0.593	0.673	0.715	0.797	0.669	0.073	F	F
U108	0.500	0.500	0.533	0.571	0.518	0.025	UP	UP
U109	0.613	0.713	0.753	0.828	0.683	0.100	F	F

*F = Fail, P = Pass, UG = Unresolved, good evidence, UP = Unresolved, poor evidence

Table 5-6
Classification of FGWL Tanks Based on the L_B Evaluation

Screening Group	Median, q_{50}	q_{95}
Pass	2	2
Fail	10	16
Unresolved, Good Evidence	9	3
Unresolved, Poor Evidence	4	4
Total	25	25

Table 5-7
Evaluation of FGWL Tanks Not Flagged by Whitney Based on L_B

Tank	Centroid Quantile				Mean	Std. Deviation	Classification	
	0.25	0.50	0.75	0.95			q_{50}	q_{95}
AX101	0.356	0.500	0.500	0.562	0.457	0.076	UG	UG
S112	0.202	0.228	0.235	0.333	0.218	0.076	P	P
SX109	0.500	0.500	0.500	0.500	0.500	0.000	UP	UP
T110	0.274	0.286	0.310	0.405	0.292	0.065	P	P

5.2.3. L_B for Non-FGWL Tanks Flagged by Whitney. Table 5-8 lists the L_B evaluation results for the additional 37 tanks flagged by Whitney not currently on the FGWL. As noted above, 24 of these tanks also were placed on the Barton list and considered for addition to the FGWL. Table 5-9 shows the number of tanks in each group at the median and 0.95 quantile for the centroid measure. At the median, only roughly 10% of the tanks flagged by Whitney fail the current evaluation. Note that this is also true for the more restricted Barton list. At q_{95}, these fractions increase to 0.3 and 0.38, respectively.

Eleven tanks are classified as UP for the non-FGWL set. Ten of these are also on the smaller Barton list. Table 5-10 shows the sensors available and the number of intervals used to calculate P_S. Tanks in this table fall into two categories: those that only have MT and NL data or those with a relatively small number of FIC intervals together with both MT and NL data or with just NL data, also in small quantities. For all of these combinations, it is difficult to generate a non-zero DOM in any likelihood set other than {Unresolved}. In fact, non-zero membership in the "likely" sets can occur only if P_S, R^2, and S are all in good agreement. R^2 must have some degree of membership in {High}, and S must have membership in {Very Negative}. This requirement was not met for any of the tanks listed in this table. The ability to qualify evidence and to avoid classifying tanks as failing the screen at high quantiles when the evidence is poor is an important capability of the AR method.

5.3. Summary

In this section, we have applied the evaluation algorithm using real tank data. A complete evaluation using the entire expert system was carried out for two tanks. The interaction of the predictor, enabler, and indicator likelihoods provides important information on how the various perspectives on tank behavior influence the final conclusion. MC simulation was used to sample the body of evidence and provide statistics for estimating the confidence one can have in the results. Both the membership vector and the centroid measure provide valuable information about the flammable gas characteristics of a tank. We also compared our results with existing WHC screening conclusions. Although the body of evidence available in both cases is essentially identical, the techniques used to process the evidence, and therefore the conclusions, differ. An important difference is that the AR model is designed to classify tanks as unresolved if the evidence quality is poor or the inferences are contradictory. In the evaluation of the Barton list using the L_B submodule alone, the current judgment is that the evidence for a number of tanks is insufficient to reach a definitive conclusion.

Table 5-8
Results for L_B for the Tanks Flagged by Whitney Not on the FGWL from Monte Carlo Simulations with 2000 Trials

Tank	Barton List	Centroid Quantile				Mean	Std. Deviation	Classification	
		0.25	0.50	0.75	0.95			q_{50}	q_{95}
A103	Yes	0.500	0.533	0.606	0.752	0.554	0.105	UG	F
AW104	Yes	0.444	0.500	0.517	0.602	0.465	0.121	UG	F
AY101	Yes	0.310	0.423	0.452	0.491	0.377	0.113	UG	UG
BX107	Yes	0.500	0.569	0.698	0.729	0.592	0.093	UG	F
BY101	Yes	0.500	0.500	0.500	0.500	0.500	0.000	UP	UP
BY102	Yes	0.500	0.500	0.500	0.537	0.505	0.017	UP	UP
BY103	Yes	0.500	0.500	0.500	0.519	0.503	0.013	UP	UP
BY105	Yes	0.500	0.500	0.500	0.500	0.500	0.001	UP	UP
BY109	Yes	0.548	0.671	0.778	0.907	0.665	0.152	F	F
C104	Yes	0.414	0.500	0.514	0.578	0.462	0.101	UG	UG
C107	Yes	0.500	0.500	0.551	0.706	0.527	0.075	UG	F
S101	Yes	0.500	0.500	0.500	0.504	0.501	0.003	UP	UP
S103	Yes	0.605	0.687	0.782	0.874	0.696	0.108	F	F
S105	Yes	0.257	0.289	0.289	0.304	0.273	0.032	P	P
S106	Yes	0.691	0.825	0.925	1.000	0.795	0.158	F	F
S107	Yes	0.500	0.534	0.622	0.732	0.560	0.105	UG	F
S109	Yes	0.154	0.230	0.256	0.354	0.198	0.108	P	P
TX102	Yes	0.500	0.500	0.500	0.519	0.501	0.006	UP	UP
TX111	Yes	0.500	0.500	0.500	0.500	0.500	0.000	UP	UP
TX112	Yes	0.500	0.500	0.500	0.539	0.504	0.013	UP	UP
TX113	Yes	0.500	0.500	0.500	0.500	0.500	0.000	UP	UP
TX115	Yes	0.500	0.500	0.500	0.500	0.500	0.000	UP	UP
U102	Yes	0.500	0.510	0.552	0.599	0.528	0.037	UG	UG
U106	Yes	0.399	0.500	0.534	0.647	0.467	0.114	UG	F
AP105	No	0.420	0.500	0.500	0.521	0.462	0.056	UG	UG
AP107	No	0.417	0.500	0.500	0.500	0.460	0.054	UG	UG
AW103	No	0.500	0.500	0.500	0.579	0.492	0.058	UG	UG
AW106	No	0.369	0.439	0.461	0.527	0.409	0.099	UG	UG
AZ101	No	0.162	0.396	0.421	0.500	0.328	0.159	P	UG
BX104	No	0.431	0.500	0.539	0.658	0.486	0.101	UG	F
BX112	No	0.500	0.502	0.595	0.713	0.543	0.091	UG	F
C105	No	0.378	0.476	0.486	0.496	0.426	0.085	UG	UG
T107	No	0.447	0.479	0.479	0.500	0.451	0.059	UG	UG
T111	No	0.407	0.500	0.500	0.525	0.450	0.082	UG	UG
TX107	No	0.370	0.492	0.500	0.500	0.440	0.073	UG	UG
TY102	No	0.351	0.461	0.500	0.528	0.423	0.099	UG	UG
TY103	No	0.500	0.500	0.500	0.518	0.502	0.009	UP	UP

Table 5-9
Classification of Non-FGWL Tanks Flagged by Whitney Using L_B Evaluations

Group	Non-FGWL Median	Non-FGWL q_{95}	Barton List Median	Barton List q_{95}
Pass	3	2	2	2
Fail	3	11	3	9
Unresolved, Good Data	20	13	9	3
Unresolved, Poor Data	11	11	10	10
Total	37	37	24	24

Table 5-10
Available Sensors and Intervals Used for Tanks Classified as UP in the L_B Evaluation

Tank	Intervals Used FIC	Intervals Used Manual Tape	Intervals Used Neutron Log
BY101	0	23	8
BY102	0	82	7
BY103	0	75	9
BY105	0	67	8
S101	29	3	8
TX102	0	11	7
TX111	0	16	8
TX113	0	16	8
TX115	0	15	8
TY103	20	0	8
U102	21	0	8

6.0. CONCLUSIONS

In this report, we have proposed an approach to screening waste tanks for a significant quantity of retained gas that differs considerably from previous methods. Our approach centers on an inductive logic structure that specifies how the evidence collected on the tank is combined to yield an estimate of the likelihood that the tank contains a significant quantity of retained gas. The evidence used in our model includes quantitative measurements of waste and tank parameters as well as qualitative judgments concerning data quality. These data were evaluated through logical rule bases to produce inferences about gas retention. Typically, several pieces of related data are combined to make a number of preliminary inferences about gas retention likelihood. All of the preliminary gas retention likelihood inferences are aggregated to produce a final likelihood inference that considers the relative qualities of the different data.

The idea behind our approach is the emulation of expert judgments. When asked to produce an estimate of flammable gas retention likelihood, an expert would examine all the data he felt was applicable. The expert would group the data into manageable segments from which he could make inferences about gas retention. He then would weigh these inferences against each other, noting supporting and contradictory inferences. He also would consider the relative strength and quality of the different data and the inference models used to combine those data. Lastly, he would make a final determination of gas retention likelihood.

Our method emulates an expert by first constructing a logic structure that models how the expert collects, collates, and combines pieces of data to make an inference. The logic structure shows which pieces of evidence are combined to make a likelihood inference. Our method also uses a set of specific implication rules that use tank data as evidence to generate inferences about gas retention. Together, the inductive logic structure and the associated inference rule sets provide an inferential structure that uses the input data to produce an output that is a measure of the likelihood of significant retained gas for any given tank.

In practice, the inductive logic structure and the rule sets should be constructed using the knowledge of a group of flammable gas tank experts. The experts determine what data should be combined to make individual inferences about flammable gas likelihood and how the individual inferences are weighted to account for mutual contradiction/support and relative data quality. In this study, one of the developers of the methodology was used as the expert source for the preliminary demonstration study so that the extra complication of expert elicitation was not added to the developmental aspects of the work. Much of the logic structure for combining evidence was based on the work of Hodgsen (1995). Thus, this study represents an example of the techniques, not a final product.

The screening approach in this study differs from past approaches in several important ways. Our approach explicitly defines the logic to be used during the screening process, whereas some past screening approaches have not defined all the logic used in their analyses explicitly. This provides a traceable path from the flammable gas likelihood back to the input data used in the inferences. The approach includes both stochastic variation in measured quantities and qualitative judgments with their associated fuzziness. This allows us to use more of the available data. Past approaches typically have used only stochastic variables in their formal analysis and have either ignored or treated qualitative data in an *ad hoc* manner. To compensate for the use of less data, past approaches typically have been very conservative in their judgments and have tagged tanks as gas-retaining based on very little and often questionable data.

The approach presented here explicitly differentiates between contradictory and poor data. With this method, poor quality or sparse data provide an indication that the gas likelihood is unresolved because of poor data. If the data are relatively good but show ambiguity in their results, then the method returns a verdict of unresolved with good data. This distinction is important in determining how to resolve the ambiguity for a particular tank. For example, collecting more of the existing type of data or using better instruments may be alternative approaches for determining flammable gas retention for a tank that is "unresolved—poor data." On the other hand, completely new types of data may be required to determine flammable gas retention for a tank where the existing data are of good quality but where the gas retention likelihood is unresolved because of contradictions between different sets of data.

The output of this analysis is a clear distillation of the input data and the logic structure that relates the various evidence. This output carries a considerable amount of both stochastic and qualitative information about the tank. The information can be compressed to a point-value statistic or examined as a distribution when screening for the presence of retained gas.

The approach used in this study has a sound mathematical basis in the theory of approximate reasoning. AR techniques are invoked to propagate tank evidence through inference models linked by the inductive logic structure. Flammable gas screening is a natural area for applying these techniques because of the inherent imprecision and the importance of qualitative judgments in this analysis. AR is a mathematical field with firm theoretical foundations and is an active area of ongoing research and expansion in both theory and application. We believe this application to be on the leading edge of the envelope for the AR field.

In this study, the power and relative simplicity of the AR approach is demonstrated by applying it to two problems: a complete evaluation of two tanks using a diverse body of evidence and the evaluation of tanks currently on the FGWL along with additional candidate tanks using the barometric pressure logic structure only. These analyses use actual tank data as input for our example inductive logic structure and inference rule set. The results demonstrate the potential for reducing unnecessary conservatism and identifying tanks with unresolvable status given the available evidence. The results also show how, using AR techniques, considerable information on the uncertainty of both evidence and knowledge of the experts can be retained in the final likelihood inference.

A new and powerful analytical approach to flammable gas screening has been demonstrated successfully in this study. The next step in the development is to construct a logic structure and rule sets based on a larger group of experts. This can be done with the techniques used here. It is our expectation that the larger group of experts would identify more data sets from which gas likelihood can be inferred. We also might expect some of the current logic to be modified to better capture the experts' judgment concerning relative data quality and imprecision in qualitative judgments. The existing study has demonstrated all the important components of an approximate reasoning approach to flammable gas screening and only awaits the addition of further experts to the knowledge base for full implementation.

APPENDIX A

BASIC CONCEPTS OF APPROXIMATE REASONING

The FGWL screening method described in this report is an application of approximate reasoning (AR). This appendix provides a discussion of the concepts relevant to AR model development and shows how the approach is based on set theory, classical predicate logic, and its extension, fuzzy logic. We examine the extension of these concepts to construction of logic trees in forward-chaining rule-based expert systems. We discuss the techniques available to evaluate the confidence one might have in the output of such an expert system and show how these results are related to more conventional expert judgment. This appendix also provides an introduction to the set-theoretic notation used throughout the report.

AR is designed to provide a structure for making inferences under conditions of ambiguity and imprecision; that is, in situations where one must resort to expert judgment and where assuring consistent evaluation is difficult (Zadeh 1976, 1975a, 1975b). Consider for a moment the type of expert-based evaluation required to reach some conclusion about a complex issue. Figure A-1 shows one possible, greatly simplified, logic structure that an expert may use. The expert is provided with some body of knowledge about the problem. He then performs some internal qualification of this data and groups it into relevant sets that are used as input for various internal models that the expert has developed through his experience. Note that the body of evidence may include the results of detailed code calculations so the expert's internal model may include his assessment of the applicability of the code models or the confidence he has in the input for the problem at hand. The expert then performs some sort of weighting of the results from his model and attempts to reach a tentative conclusion. This can be an iterative process. If a conclusion can be reached, the expert also is normally asked to assign some confidence to his conclusion. This can be expressed as a probability estimate or may be couched in more qualitative terms. We refer to the conclusion along with some assertion of confidence in that conclusion as the expert judgment.

Various methods are available for expert elicitation; the reader should refer to Myers and Booker (1991) for a detailed discussion. Regardless of the elicitation method, a common problem arises when the expert is asked to make a summary judgment about a complex issue. In this case, it is extremely difficult to frame the questions so that the elicitor can document the expert's internal models and be sure that the specified boundary conditions are being used. In other words, we may have an expert opinion but not be sure that it is really for the problem of interest and not necessarily know how it was arrived at. This difficulty becomes more acute as the complexity of the problem increases and as more experts are interviewed. In the latter case, it is hard to confirm that all the experts have used a common set of data and assumptions. There are also potential problems in aggregating the responses. Certain group techniques such as Delphi attempt to get around these obstacles. However, it is our judgment that this cannot be done in a robust and defensible manner for FGWL screening because of the large number of tanks to be evaluated and the vast body of evidence that must be considered.

In an AR approach, the underlying framework is a logic structure that is used to order a chain of inferences about a body of evidence. The elements of evidence are described using natural language expressions that are consistent with the words an expert would use. These expressions are called linguistic variables. Examples of the use of linguistic variables for the flammable gas body of evidence are statements such as the following.

The barometric pressure-level fluctuation correlation is *very strong*
The radiolytic gas generation is *quite low*

Figure A-2 shows a simplified schematic for simulating expert judgment using an AR-based approach. We start with the same body of evidence available to the expert. This evidence serves as the *primary inputs* to the evaluation algorithm. The evidence is converted into linguistic variables, and to draw consistent inferences, these variables are treated as sets. In this form, the formal mathematics of set

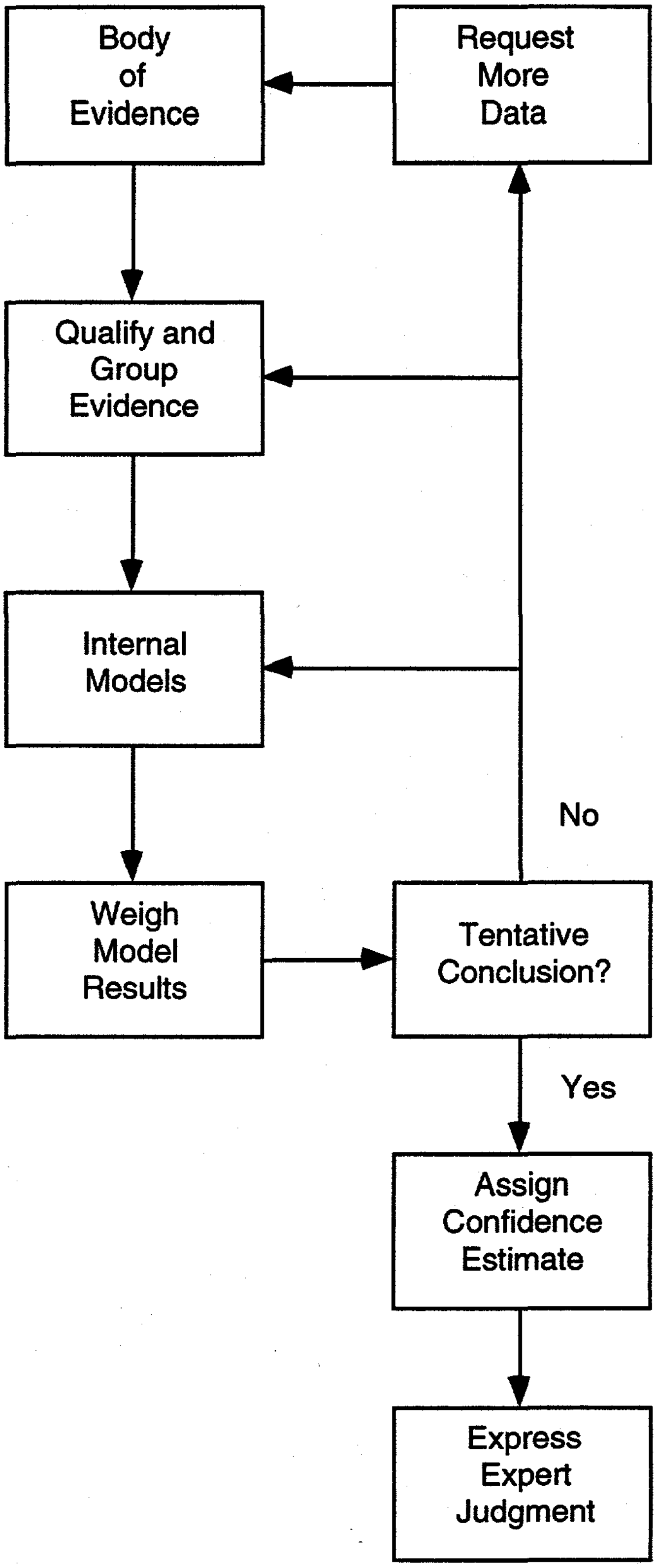


Fig. A-1. Simplified schematic for expert judgment process to screen a tank.

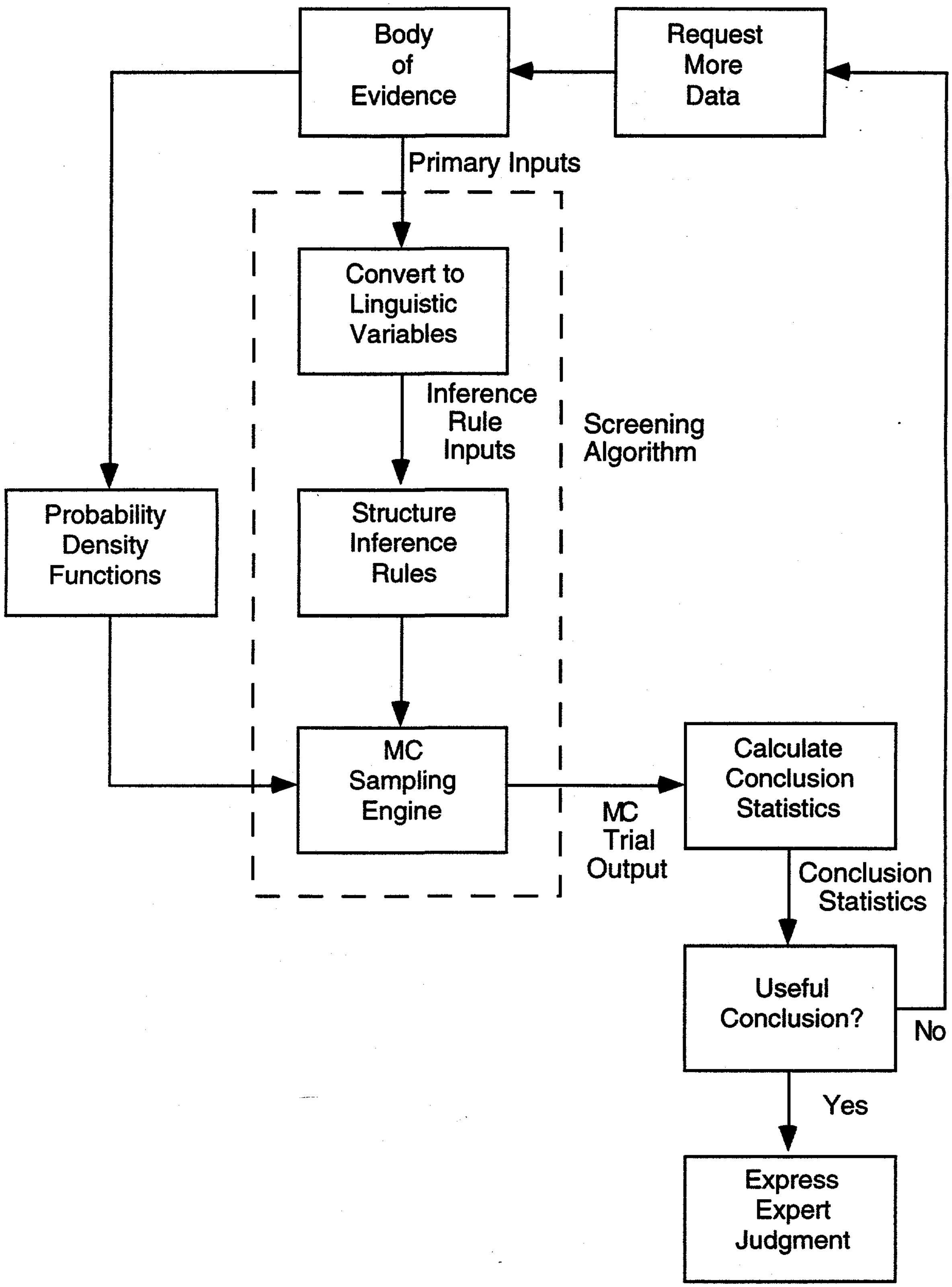


Fig. A-2. Simplified schematic for an approximate reasoning expert system to screen a tank.

algebra are available, and we can apply the formal logic operation of implication that lies at the heart of any inference model. The AR model is a set of structured inference rules generated by a logic structure that emulates the internal processes used by an expert in evaluating evidence and arriving at a conclusion. The aggregate output of the inference module is a conclusion.

Monte Carlo sampling is used to obtain some measure of the confidence associated with this conclusion. The body of evidence contains random variables that are represented by probability distribution functions as well as "raw" expert judgment that may be aggregated using statistical models. Monte Carlo sampling allows one to associate statistical measures with the tentative conclusion and thus render an AR-based judgment that is the analog of the expert's. In the remainder of this appendix, we briefly discuss the concepts used in Fig. A-2 and provide the background information needed for the development of a detailed evaluation.

1.0. LINGUISTIC VARIABLES AND FUZZY SETS

The use of linguistic variables is best illustrated by example. Here we present a modified discussion of that in Ross (1995). Suppose that we wish to group a collection of people according to their heights. We define the collection of heights as the universe of discourse, H, and any individual height by h. In set theoretic notation $h \in H$, or h is an element of H. For our grouping, we wish to use the linguistic descriptions "Tall," "Average," and "Short." That is, we define three sets {Tall}, {Average}, and {Short} that are contained within H Suppose that we know the heights fairly accurately, for example, to the nearest 1/4 in. We can define the characteristic heights for each set formally, for example,

$h \in \{\text{Tall}\}$ iff $h \geq 6' \, 0'$
$h \in \{\text{Average}\}$ iff $5' \, 6'' < h < 6' \, 0'$
$h \in \{\text{Short}\}$ iff $h \leq 5' \, 6''$

Let the set D defined on H be D = {5'2", 5'4", 5'7", 5'10", 5'11-3/4", 6'1/4", 6"5"}. Then the intersection of D with the three sets used to group heights is $D \cap$ {Short} $\in$ {5'2", 5'4"}, $D \cap$ {Average} $\in$ {5'7", 5'10", 5'11-3/4"} and $D \cap$ {Tall} $\in$ {6'1/4", 6'5"}. Note that for any h, we can say definitely whether or not it is contained within the intersection of D and one of our three sets. Sets like this are referred to as crisp. Membership in a crisp set, Λ, can be expressed using the characteristic function χ:

$$\chi(x,\Lambda) = 1 \text{ if } x \in \Lambda \qquad \text{(A–1)}$$
$$\chi(x,\Lambda) = 0 \text{ if } x \notin \Lambda$$

It can be easily seen that crisp sets are not well-suited to dealing with ambiguity and imprecision. For example, suppose that H is the set of heights of all professional baseball or football players. The definition for the linguistic "Tall" given above might be reasonable for baseball players but not for football players. It would be possible to have two sets of definitions, but this is not always practical. Also in our example, h = 5'11-3/4" $\in$ {Average} while h = 6'1/4" $\in$ {Tall}. This occurs when a crisp threshold is used. In practice, we often want to avoid such sharp distinctions and might want to think of both people as "rather tall." Here again we could define a new set, for example, between 5'10' and 6'2", but the basic problem remains. Imprecision also arises when our judgment of the heights is more subjective, as when we classify by eye. It is unlikely that we could differentiate between two people very close to 6'0" in height. In this case, it is clear that our crisp sets do not accurately reflect the actual judgments being used to assign each person to membership in the three sets.

To avoid these problems, Zadeh introduced the concept of fuzzy sets (Zadeh 1965). An element can have partial membership in a fuzzy set. In our example, when using fuzzy sets, h can belong to both {Tall} and {Average}, h $\in$ {Tall} and h $\in$ {Average}. Membership functions such as those shown in Fig. A-3 for the example problem normally are used to convert a quantitative value to membership in fuzzy sets. This is referred to as fuzzification.. The degree of membership (DOM) for an object x in a set Λ is denoted as $\mu(x,\Lambda)$ and by definition $0 \leq \mu(x,\Lambda) \leq 1$. In our example for h = 6'1/4", μ(6'1/4",Tall) = 0.55

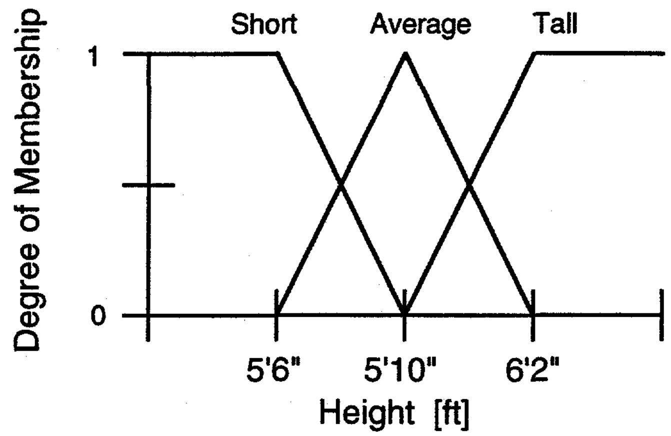


Fig. A-3. Membership functions for height for the example problem.

and μ(6′5″,Tall) = 1.0. This approach explicitly provides for ambiguity. Suppose that h has DOMs μ(t,Short) = 0, μ(t,Average} = 0.4 and μ(t, Tall) = 0.6. We will denote this by the vector γ(h): {0, 0.4, 0.6} and use the notation h∈ { Short, Average, Tall} to indicate the fuzzy sets in which h can have membership. This class of sets, { Short, Average, Tall} is defined on the universe of discourse H, which in this case is all possible heights for a person. The vector γ(h): {0, 0.4, 0.6} corresponds to someone approximately 6′1″ tall. We can think of this individual as "rather tall." Note that the DOMs change smoothly as we change the height under consideration—in this case, as the height decreases, membership in {Tall} goes down and membership in {Average} increases. Below 5′10″, membership in {Short} becomes non-zero, whereas μ(h : h≤5′10″, Tall) = 0. Because there are no crisp thresholds, the combination of linguistic variables and fuzzy sets is a powerful tool for drawing inferences.

Several additional technical points should be mentioned briefly.

1. All of the operations defined for crisp sets also are defined for fuzzy sets. For example, the union operation, $\Lambda \cup \beta$ is defined for fuzzy sets as $x \in \Lambda \cup \beta = \max(\mu(x,\Lambda), \mu(x,\beta))$. This follows the crisp rule $x \in \Lambda \cup \beta = \max(\chi(x,\Lambda), \chi(x,\beta))$. Similarly, the intersection is $x \in \Lambda \cap \beta = \min(\mu(x,\Lambda), \mu(x,\beta))$.
2. One important difference between crisp and fuzzy sets is the law of the excluded middle. For crisp sets, the intersection of a set and its complement is empty: $\Lambda \cap \Lambda' = \varnothing$. However, for fuzzy sets, this is not true: $\Lambda \cap \Lambda' \neq \varnothing$. A DOM μ(x,Λ) = 0.5 means that x has equal membership in Λ and in its complement.
3. All of the membership functions used here are either triangular or trapezoidal in shape. This satisfies a requirement that they be convex. Also, in this report, membership functions are defined so that any element x for which membership functions are defined explicitly for a universe of discourse $x \in \{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, ...\}$ can have membership in at most two of the sets defined on this universe. This is definitely *not* a general requirement for working with fuzzy sets and was adopted here for simplicity. For further details, refer to Ross (1995) or Kosko (1992).

2.0 FUZZY SETS AND INFERENCE

Sets and set operations combined with the elements of classical logic provide a formal structure for generating inferences from evidence. This formal structure forms the theoretical basis for the inferential rules that drive the FGWL algorithm. Let P be a proposition that is true if $x \in \Lambda$ and let Q be a proposition that is true if $x \in \beta$, where both are defined on the universe of discourse $x \in X$ and where, for the moment, Λ and β are crisp sets. The truth value of P, T(P) = 1 if $x \in \Lambda$ and T(P)=0 if $x \notin \Lambda$. Recall that the standard logical connectives are the following.

$$\text{Disjunction, } P \vee Q - x \in \Lambda \text{ or } x \in \beta \tag{A-2a}$$
$$\text{Conjunction, } P \wedge Q - x \in \Lambda \text{ and } x \in \beta \tag{A-2b}$$
$$\text{Negation, } \overline{P} - x \notin \Lambda \tag{A-2c}$$
$$\text{Implication, } P \rightarrow Q - x \notin \Lambda \text{ or } x \in \beta \tag{A-2d}$$
$$\text{Equivalence, } P \leftrightarrow Q - (x \in \Lambda \text{ and } x \in \beta) \text{ or } (x \notin \Lambda \text{ and } x \notin \beta) \tag{A-2e}$$

The truth of the expression $T(P \vee Q) = 1$ if either $T(P)$ or $T(Q) = 1$; that is, $T(P \vee Q) = \max(T(P), T(Q))$. Similarly for the conjunction connective, $T(P \wedge Q) = \min(T(P), T(Q))$. Because P and Q can be combined only in four ways, it is easy to construct a truth table to show all of the possible truth values for each connective. This is shown in Table A-1.

In the remainder of this appendix, we will be concerned primarily with implication, $P \rightarrow Q$. Note from Table A-1 that $T(P \rightarrow Q) = \max(T(\overline{P}), T(Q))$. That is, the expression is true except in the case where P is true and Q is false; by definition, a true hypothesis cannot imply a false conclusion. It is not necessary that Λ and β be defined on the same universe X. The most common situation is $x \in \Lambda$ defined on X and $y \in \beta$ defined on Y. In this case $P \rightarrow Q = \overline{\Lambda} \cup \beta$. It can be shown (Ross 1995) that this implication on two universes of discourse can be written in the form of a relation:

$$R = (\Lambda * \beta) \cup (\overline{\Lambda} * Y) \equiv \text{If } \Lambda, \text{ Then } \beta \,, \tag{A-3}$$

where * is the Cartesian product of sets. This identity is very powerful. Consider our example for heights. Suppose we have found a correlation between height and the sports that people play. To begin, we assert that

Implication 1: If a person's height is "tall" then he plays "basketball or football."

Here Λ = {Tall} and β is the set of people that play basketball or football, which we denote β = {BF} defined on the universe S. From our study of height and sport, we also determine that two other sets of sports are played—soccer or racquetball (SR) and golf or tennis (GT)—and that neither of these sets of sports is played by people over 6 ft tall. We can show the relation R graphically as in Fig. A-4. The two shaded areas in the figure represent the region where the implication above is true—one where both $x \in \Lambda$ and $y \in \beta$ is true and the other where $x \notin \Lambda$. Another way to see that this is really the region in X * Y where the implication is true is to construct a table such as Table A-2. Here the terms in parentheses are the truth values for the height and sport group, respectively, and the truth value of the implication for this pair is taken from Table A-1. A value of (1,0) means that the characteristic function for height $\chi_\Lambda = 1$ and the characteristic function for $\chi_\beta = 0$. That is, P is true because $x \in$ {Tall} and Q is false because $y \notin$ {Basketball or Football}. The shaded region again is the region where the implication is true, and it is clear that the result is the same as in Fig. A-4.

Table A-1
Truth Table for Logical Connectives in Eq. (A-2)

P	Q	$\overline{P}$	$P \vee Q$	$P \wedge Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T(1)	T(1)	F(0)	T(1)	T(1)	T(1)	T(1)
T(1)	F(0)	F(0)	T(1)	F(0)	F(0)	F(0)
F(0)	T(1)	T(1)	T(1)	F(0)	T(1)	F(0)
F(0)	F(0)	T(1)	F(0)	F(0)	T(1)	T(1)

Table A-2
Truth Table for Implication #1 with T(P) = 1

T = "Λ"	1	(1,0) F	(1,0) F	(1,1) T
A	0	(0,0) T	(0,0) T	(0,0) T
S	0	(0,0) T	(0,0) T	(0,0) T
		0	0	1
		SR	GT	BF = "β"

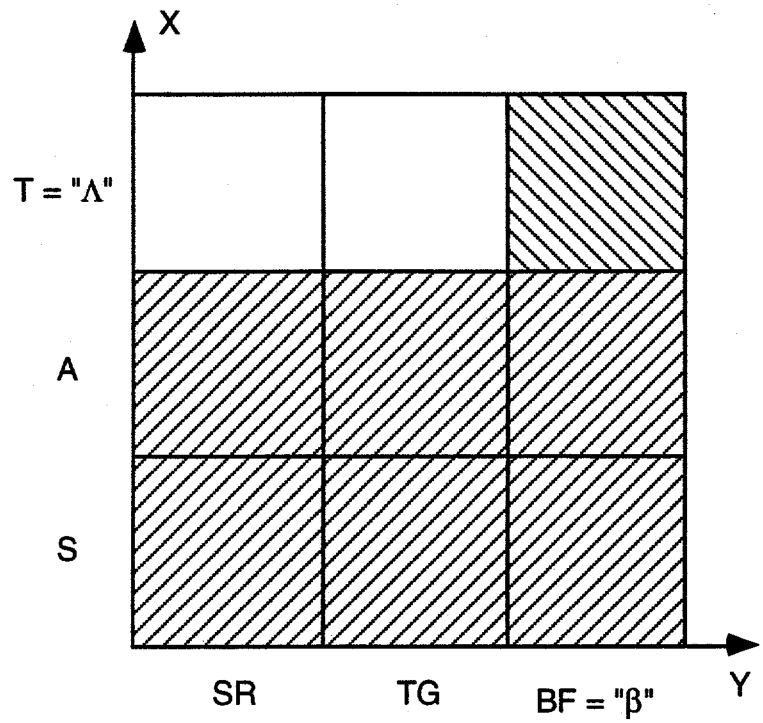


Fig. A-4. Cartesian representation for implication. In the shaded region, the implication $P \rightarrow Q$ is true.

The relation R for our example can be shown to be given by the rule

$$\chi_R(x,y) = \max\{\min\{\chi_\Lambda(x), \chi_\beta(y)\}, \{1 - \chi_\Lambda(x)\}\} . \tag{A-4}$$

This is referred to as the min-max rule and may be shown in matrix form as

$$R = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} , \tag{A-5}$$

where the min term in Eq. (A-4) gives the value in the upper right-hand element and the (1- $\chi_\Lambda(x)$) term determines the rest of the matrix. The similarity to both Fig. A-4 and Table A-2 is apparent. The significance of this representation is that if we now have a new set height, Λ', we can use the operator R to get β', the vector of sports that Λ' can play without violating the proposition, directly by composition, $\beta' = \Lambda' \circ R$. Suppose that in this case Λ' = {Short}, which we denote by {1,0,0}. Then, again without going into detail, β' = {1,1,1}. This vector states that regardless of what sport the person who is short plays, the implication is true. The validity of this statement can be verified from Table A-3. Because our rule only tells us what sports a tall person plays, it is hardly surprising that we can make no inference about what sports a short person plays.

When rules are defined to cover more of the sets in the universe, more specific inferences can be made. If we make the following implications in our example:

Implication 2: If a person's height is "average" then he plays "golf or tennis"
Implication 3: If a person's height is "short" then he plays "racquetball or soccer"

then more specific inferences are possible. We construct relations R_2 and R_3 for both of these implications that are similar to Eq. (A-5). Suppose further that our proposition P about height is {0,1,0}, which corresponds to the linguistic proposition

Bob is of average height.

If we then evaluate the implications for Q (the sport played) using each of our rules, the result is

Implication 1—{1,1,1}
Implication 2—{0,1,0}
Implication 3—{1,1,1}

The first and third result occur for the same reason as noted above—if a person is not a member of the height set for which the implication applies, we can make no positive statement about what sport he plays. However, the second rule says something entirely different. We can express the result as

Bob is of average height.
Average height people play golf or tennis.
Bob plays golf or tennis.

Assertions of this form are referred to as *modus ponens*. In symbolic form, this assertion can be written as

$$(\Lambda \wedge \Lambda \rightarrow \beta) \rightarrow \beta \ . \tag{A-6}$$

Table A-3
Truth Table for $\beta' = \Lambda' \circ R$

T	0	(0,1) T	(0,1) T	(0,1) T
A	0	(0,1) T	(0,0) T	(0,1) T
S	1	(1,1) T	(1,1) T	(1,1) T
		1	1	1
		SR	GT	BF

This statement is a tautology: "If Λ and Λ implies β then β." *Modus ponens* is the basis for forward-chaining, rule-based expert systems. The evaluation methodology used in this report is exactly such an expert system. For this reason, all of the rules in the discussion that follows will be of the form in Eq. (A-6).

A second characteristic of the rules used in our logic model is that the antecedent is always a compound statement, giving a relation of the form

$$\{(\Lambda \wedge \beta) \wedge (\Lambda \wedge \beta) \mapsto K\} \rightarrow K . \tag{A-7}$$

In our example, suppose that we have determined that knowing a person's weight helps us differentiate between the two sports that a person of a given height plays. That is, there are now compound rules of the following form.

If a person is short and heavy then the person plays soccer
If a person is short and medium weight then a person plays soccer
If a person is short and light then a person plays racquet ball
If a person is average and light then a person plays tennis
If a person is average and medium then a person plays tennis
If a person is average and heavy then a person plays golf
If a person is tall and light then a person plays basketball
If a person is tall and medium then a person plays football
If a person is tall and heavy then a person plays football

This complete set of rules, which we refer to as a rule base, then can be expressed in compact form as shown in Table A-4. For example, the upper right-hand cell represents the rule

If a person is tall and heavy then the person plays football.

Let the truth values be {0,0,1} for height and {0,0,1} for weight, then Table A-4 is used to generate Table A-5. As before, we find the implication by application of the min-max rule and see that only the statement directly above has a non-zero characteristic function. All of the rule sets that follow will be in the form of *modus ponens* with a compound antecedent.

Table A-4
Compact Form for *Modus Ponens* Rule Set for Height and Weight Inference

T	B	F	F
A	T	T	G
S	R	S	S
	L	M	H

Table A-5
Evaluation of *Modus Ponens* Rule Set for Height and Weight Inference

T	1	B (1,0)	F (1,0)	F (1,1)
A	0	T (0,0)	T (0,0)	G (0,1)
S	0	R (0,0)	S (0,0)	S (1,0)
		0	0	1
		L	A	H

Thus far, the discussion of inference has been restricted to crisp sets. All of the discussion above applies equally well for fuzzy sets except that instead of using crisp characteristic functions for set membership, one uses the fuzzy membership function μ. That is, we interpret T(P) to be $\mu(x,\Lambda)$ and substitute μ for χ in Eq. (A-2). To carry our example further, we would need to define membership functions for weight as well, but we will just say that the universe of discourse W is represented by {Light}, {Medium} and {Heavy} without specifying the exact functions. Now say that $\gamma(h)$: {0,.5,.5} and $\gamma(w)$: {0,.3,.7}. This is a person who is "rather tall" and "quite heavy." In the fuzzy representation, Table A-4 becomes as follows (Table A-6).

Application of the min-max rule [Eq. (A-4)] yields the following memberships for the various sports.

$\mu(R) = 0$
$\mu(S) = 0$
$\mu(T) = 0.3$
$\mu(G) = 0.5$
$\mu(F) = 0.5$
$\mu(B) = 0$

In more formal notation, $s \in$ {Racquetball, Soccer, Tennis, Golf, Football Basketball} and $\gamma(s)$ = {0,0,.3,.5, .5,0}. With fuzzy logic, the rule set generates membership in three sets, whereas only one set was non-empty in the crisp representation. We say that for this case, "four of the rules fired." Also given the somewhat artificial rule base, the judgment that a "rather tall" and "quite heavy" person would probably tend to play golf and football reflects quite well the "expert opinion" incorporated in the rules.

3.0. LOGIC MODEL DEVELOPMENT

The restriction to dual-element compound propositions means that the logic tree used to model the forward-chaining expert system will have the basic structure shown in Fig. A-5. There are always two inputs. There is also only one output, consistent with Eq. (A-5). As shown in Fig. A-6, a more complicated logic tree can be developed easily using this structure. Note that there are many inputs but only one final aggregate output. Each input that appears at the left is referred to as a primary input, I_i, and is defined on its own universe of discourse, Ψ_i. Therefore, we must determine the linguistics used to identify the fuzzy sets defined on this universe as well as the membership functions for each set. Suppose that we use m fuzzy sets to describe universe Ψ_1 and n fuzzy sets for universe Ψ_2. Then the rule table will have m x n entries. The output from the rule O_{12} is defined on its own universe, Ψ_{12}. The only restriction on the number of fuzzy sets used to represent this universe, l, is that $l \leq m \times n$.* For j primary inputs, there are j-1 rule sets required to obtain the final output. For each rule set, one must specify the

Table A-6
Evaluation of *Modus Ponens* Rule Set with Fuzzy Sets for Height and Weight Inference

T	.5	B (.5,0)	F (.5,.3)	F (.5,.7)
A	.5	T (.5,0)	T (.5,.3)	G (.5,.7)
S	0	R (0,0)	S (0,.3)	S (0,.7)
		0	.3	.7
		L	M	H

*Otherwise there are sets in the output universe that are unreachable.

corresponding $m_i + n_i$ membership functions as well as $m_i \times n_i$ individual rules and define l fuzzy sets on the universe of the output from the rule. Thus, for a reasonably complex problem, say j = 50, and with m, n and l equal to 4, there are 400 membership functions, linguistics for 200 output fuzzy sets, and 784 individual rules to specify. Although this represents a significant amount of work, the benefit is that one is forced to describe quite clearly how the evaluation is being done.

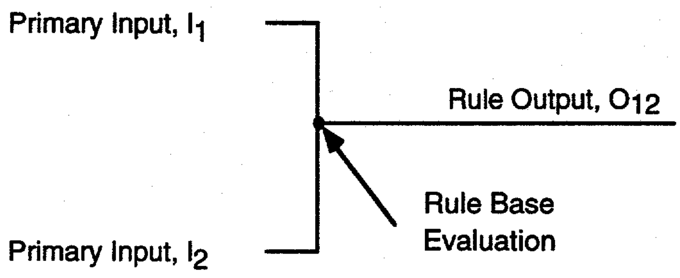


Fig. A-5. Basic structure for forward-chaining expert system.

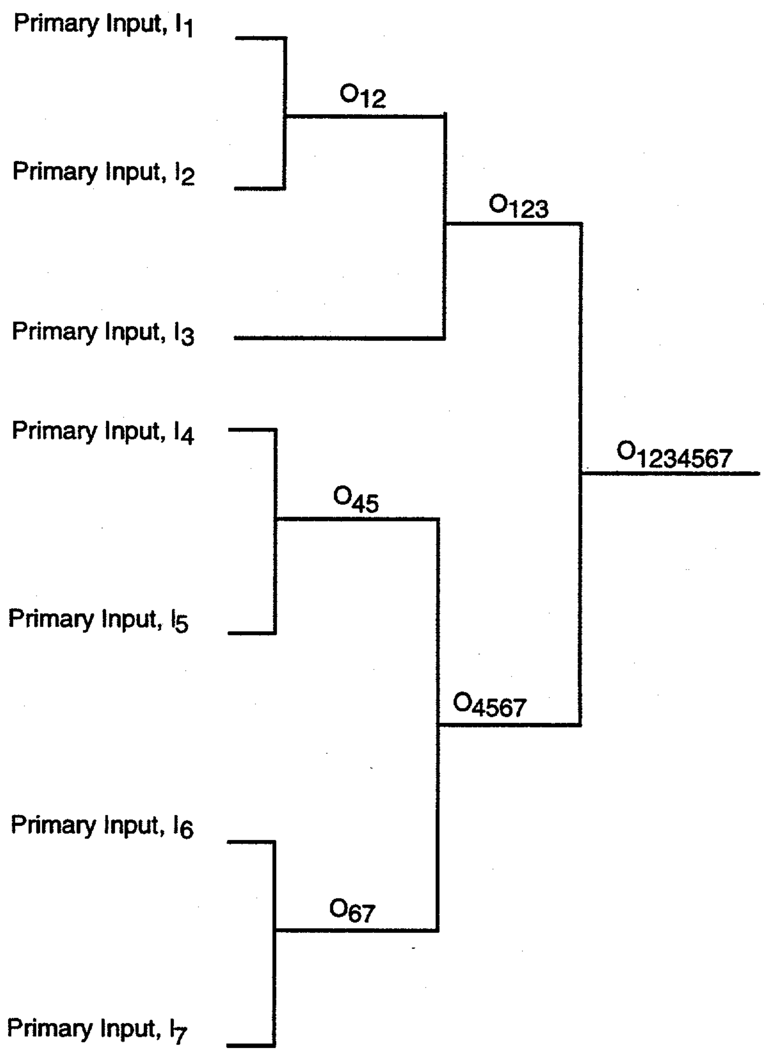


Fig. A-6. Illustrative logic tree with seven primary inputs and six rule sets.

For our simple example, the logic tree is shown in Fig. A-7 and all of the quantities required to perform a complete evaluation have been defined already. We only need to specify the numerical values for the two primary inputs, which are the height and weight of the individual for whom the evaluation is to be made. Suppose that we wish to add another rule to the model, possibly by using a person's age to make some prediction about how well a sport is played. The output from the first rule becomes an input to rule number two along with the DOMs for the primary input for age. This illustrates how relatively simple it is to add new branches to the existing logic structure. The ability to simply prune and graft logic branches or entire modules is an important attribute of this approach.

4.0. REACHING A FINAL CONCLUSION

It is clear that in designing an expert system, one must have a good understanding of the final judgment that is needed. This means first specifying the universe of discourse for the final output. For the FGWL screening problem, we use the likelihood of a significant quantity of retained gas as the metric. The logic structure is intended to emulate the evaluation of a real expert, so it is natural to produce a judgment that is in the same terms as would be expected from the expert. There are several possibilities. The expert could be rather precise and say something like the following.

Response #1 – In my opinion there are 4000 cubic feet at STP of retained gas in the tank

or perhaps

Response #2 – It looks like there is a lot of gas in the tank

or

Response #3 – In my opinion it's quite likely that there is a significant amount of retained gas in the tank.

Of these responses, we currently believe that the third is the most useful form in which to express a judgment related to FGWL screening for several reasons. First, the judgment is expressed in terms of a likelihood. This is a common way in which expert opinion is framed and is also in a form where it can be readily understood by a decision maker. We use the expression "likelihood" in the sense that it "supplies a natural order of preference among the possibilities under consideration" (Thomas 1995). That is, something that is said to be "very likely" is understood to have a more realistic chance of happening or to occur more frequently than something that is "extremely unlikely." However, it must be emphasized that the likelihood linguistic variable is not to be confused with quantitative probability nor do we intend our use of likelihood to be associated directly with the likelihood function of probability theory. Second, the concept of likelihood leads naturally to the specification of the outputs from intermediate rules.

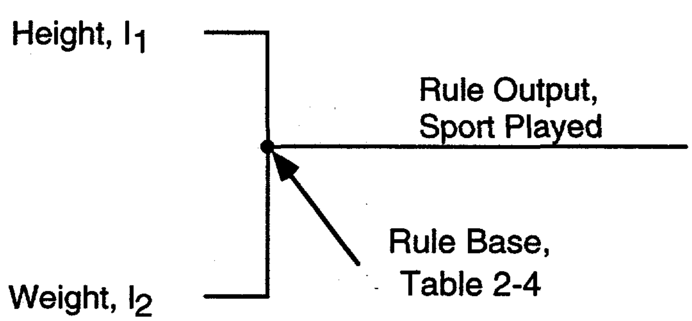


Fig. A-7. Logic tree for height-weight example problem.

Note also that in the third response above, the quantity about which the opinion is expressed is a "significant amount of gas" rather than an actual numerical value. There are two advantages to this. First, an expert will use such a formulation more frequently because it allows for a degree of ambiguity and imprecision characteristic of AR. A quantitative number is, in fact, not really of greater value given the complicated evaluation that must be performed. Second, for screening purposes, there is no need to specify an "exact" quantity of gas. If a tank fails the screen, then more detailed analysis can be done where quantities of retained gas, release fractions, and dome-space per cent of LFL are calculated based, if necessary, on further observation and testing.

In the illustrative example of this appendix, it might better reflect the true understanding of the relationship between height, weight, and sport of choice if we said the following.

If a person is *Short* and is *Light* THEN it is *Quite Likely* that the person plays racquetball.

To illustrate the application of likelihood in an evaluation, let us change our example problem slightly. Now the judgment desired is a statement about the likelihood that given a person's height and weight, he or she will play tennis. On the universe of discourse L, we define three fuzzy sets described by the linguistic variables {Quite Unlikely}, {Unresolved}, and {Quite Likely}, which we will refer to as {QU}, {U}, and {QL}, respectively. The likelihood of playing tennis then is expressed by the DOMs in these three sets, L_T: {QU, U, QL}. The set {U} is included to account for the situation where the information supplied to the rule set is so ambiguous that the best judgment is really "I don't know."

Table A-7 shows the evaluation for this case where the rules associating height and weight with the propensity to play tennis are somewhat arbitrary, but where the input pair (Average, Medium) implies a {QL} judgment and corresponds to the old rule that a person with these attributes plays tennis. The output of this rule evaluates to {.5,.5,.3}. These are the same numerical values as before, except now rather than describing the DOM in the set of people who play tennis $\mu(T) = 0.3$, we have made instead a judgment about how likely it is that a person with these vital statistics plays tennis. In this case, $\mu(QL) = 0.3$, whereas $\mu(QU) = \mu(U) = 0.5$, so our judgment might be that it is "quite unlikely" that this particular person plays tennis. Note also that the information is really not very good for making such a judgment as the membership in {U} is relatively high. Thus, we can see that some judgment about the quality of the data can be incorporated in the inference.

Table A-7
Evaluation with Fuzzy Sets of *Modus Ponens* Rule Base for Example Problem to Determine Likelihood of Playing Tennis

T	.5	QU (.5,0)	U (.5,.3)	QU (.5,.7)
A	.5	U (.5,0)	QL (.5,.3)	U (.5,.7)
S	0	QU (0,0)	U (0,.3)	QU (0,.7)
		0	.3	.7
		L	M	H

It was mentioned earlier that not only should the evaluation provide some likelihood judgment but, just as with an expert, also some assertion of confidence in the evaluation. In the example problem, we might want to evaluate the likelihood that some group of people, say the members of a health club, play tennis. Our rule base was developed with membership functions where "average" and "medium" were intended to represent the population as a whole. The members of the club probably have somewhat different height and weight characteristics. More precisely, we expect that the PDFs are different than those used in developing the evaluation model. In this simple case, we could just input the means or medians of the member distributions and get the desired result. However, this is not really practical in more complicated models where there are many primary inputs described by separate density functions. The min-max operation used in the rules is nonlinear, and to make some assertion of confidence in the conclusion from the logic structure, some knowledge of the PDF for the aggregate output is required.

Information about the statistics of the final output in a logic structure can be obtained with Monte Carlo sampling. In our example for each trial, random samples for height and weight are obtained.* These numerical values are converted to memberships in the fuzzy sets defined for both primary inputs, and the single rule base is evaluated. For each trial, the output is a vector with the DOMs in the three likelihood fuzzy sets. For example, the membership vector after trial j might be $\gamma_j(L) = \{0.2, 0.5, 0\}$. At the end of the simulation, we have N estimates for γ. It can be seen that there are three distinct PDFs associated with γ:

$$\mathrm{PDF}(\gamma(L)) = \{\mathrm{PDF}(\mu(\text{Quite Unlikely}), \mathrm{PDF}(\mu(\text{Unresolved}), \mathrm{PDF}(\mu(\text{Quite Likely})\} \ .$$

One approach to characterizing the uncertainty in L is to derive statistics from this vector. In this case, if we ask about the value of L at some quantile, q_i, associated with $\mathrm{PDF}(L_F)$ we use the vector

$$q_i^* = [\ q_i(\mu(\text{Quite Unlikely})), q_i(\mu(\text{Unresolved})), q_i(\mu(\text{Quite Likely}))\ ] \ . \tag{A-8}$$

This vector contains the DOMs at the q_i quantile for each set in the universe of discourse for L. However, note that the vector q^* is not itself the q_i quantile for L. We must specify how to process the vector to compute $q_i(L)$. A natural approach is to define $q_i(L)$ as

$$q_i(L) = \max\,[q_i(\mu(\text{Quite Unlikely})), q_i(\mu(\text{Unresolved})), q_i(\mu(\text{Quite Likely}))] \ . \tag{A-9}$$

This specification for q_i is the maximum DOM associated with the likelihood sets at this quantile. For example, suppose that at the conclusion of the Monte Carlo simulation we have the following results at the 0.9 quantile:

$$q_{90}^* = \{0.2, 0.4, 0.8\} \ .$$

This corresponds to the following sequence of statements about the likelihood that a person selected at random from the population of health club members plays tennis.

At the 0.9 quantile, the DOM in the set {Quite Unlikely} is 0.2.
At the 0.9 quantile, theDOM in the set {Unresolved} is 0.4.
At the 0.9 quantile, the DOM in the set {Quite Likely} is 0.8.

And the natural language expression associated with this result is the following.

At the 0.9 quantile, it is quite likely that a health club member plays tennis.

We refer to q^* as the membership vector measure for the likelihood statistics.

A second approach to estimating statistics for L involves calculating a measure from L_i for each trial and then obtaining an estimate for the density function of this measure in the MC simulation. The process of calculating a single measure from degrees of membership in a class of fuzzy sets is called defuzzification. Various methods exist to obtain a crisp output. We chose to use the centroid method. This is often argued to be the method most consistent with an expert's "best-estimate" estimate. To defuzzify, it is necessary to define membership functions for L. For the example, we use the membership functions in

*We ignore the obvious correlation between height and weight for this example.

Fig. A-9 to represent the likelihood judgments about tennis playing. It must be emphasized that although the functions are defined between 0 and 1, this does not mean that the fuzzy sets represent actual probabilities.** The defuzzification process is shown in Fig. A-10 for the membership vector $\gamma(L) = \{.2, .4, .8\}$. The membership functions for each set in Fig. A-9 are multiplied by the degree of membership for each set in $\gamma(L)$. The union of these weighted membership functions is a max operation that produces the outer envelope in regions where the membership functions overlap. We denote this outer envelope as M; then the centroid λ for L is found by

$$\lambda(L) = \int_0^1 x\, M(x)\, dx \Big/ \int_0^1 M(x)\, dx\ , \qquad \text{(A-10)}$$

where the integral denotes algebraic integration. For $\gamma(L) = \{.2, .4, .8\}$ the centroid is $\lambda(L) = 0.64$.

For each trial we compute the centroid, $\lambda(L)$, from the output likelihood membership vector and calculate statistics for λ from the collection of trials in the Monte Carlo simulation. We refer to this as the centroid measure and speak of centroid statistics. Suppose that the median and 0.9 quantile estimates for $\lambda(L)$ are $\lambda(q_{0.5}) = 0.58$ and $\lambda(q_{0.9}) = 0.65$ respectively. We can then make statements of form

The median centroid for the likelihood that a member of the health club plays tennis is 0.58.

or

The probability that λ (L)$\leq$ 0.65 is 0.9.

The logical natural language expression for L based on the centroid is the set in which it has the maximum DOM. For example, if $\lambda(q_{0.9}) = 0.8$, then from Fig. A-9, the likelihood set with the largest DOM is {Quite Likely}. The corresponding natural language expression associated with this is

At the 0.9 quantile it is quite likely that a health club member plays tennis..

Both the membership vector and the centroid approach to calculating statistics for L and for obtaining the associated natural language expressions are useful. The quantile vector approach provides valuable information on how often the various rules in the rule bases "fire" as the quantile changes. One disadvantage associated with this approach is that the dispersion associated with PDF(L) is not well-represented. Consider for example the vector

$q^*_{90} = [.95,.95,.96]$.

In this case, the operations above would lead to a "quite likely" result when an evaluation of "unresolved" would be appropriate. Because of the tendency for multiple elements in the quantile vector to approach 1.0 at high quantiles, this problem is commonplace. The centroid method loses information on the relative growth of membership in the likelihood sets but provides an easily understood measure for estimating statistics and can be transformed back into a natural language expression in a straightforward manner.

**There is no requirement that the memberships sum to one nor is there any intention that {Unresolved} imply equally likely.

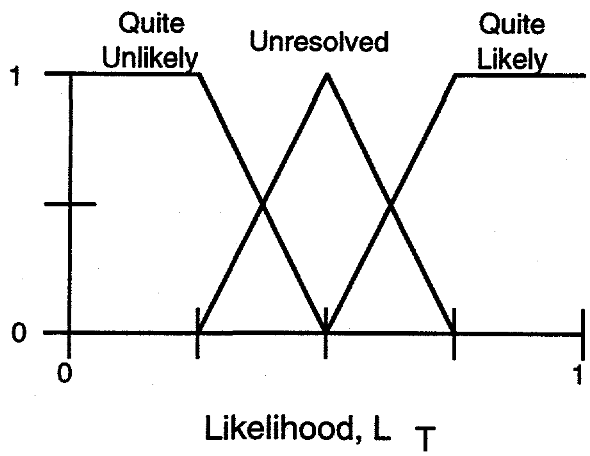


Fig. A-9. Membership functions for sets used to describe the likelihood of playing tennis.

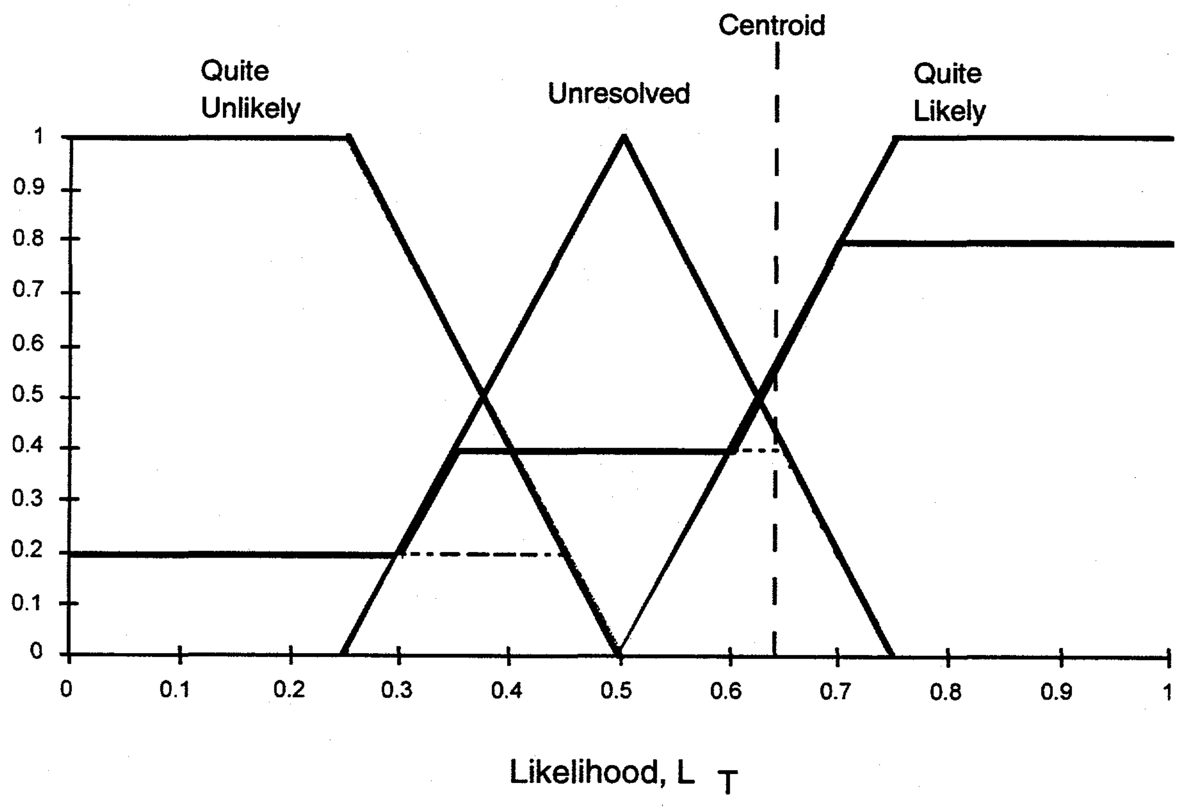


Fig. A-10. Defuzzification process for $\gamma(L) = \{.2,.4,.8\}$.

APPENDIX B

CHARACTERISTICS OF IMPLICATION RULE BASES

The FGWL screening algorithm contains a series of forward-chaining rule bases that use formal logical implication to make inferences about the likelihood of retained gas. The overall process in the AR model is to convert input measurements, estimates, calculations, or observations into likelihood estimates. The rule bases define the specific relationship between the inputs at each junction in the inductive logic structure.

All of the rule bases used in the AR model have two inputs, which are referred to as the antecedents. The output of the implication is the consequent or conclusion. The theory associated with implication rule bases is discussed in Appendix A. Every rule base can be represented as a matrix with the elements of one input set in the first column and the other in the last row. The consequent implied by the combination of any ordered pair of antecedents is given in the matrix element with the same row as the column input and the same column as the row input.

1.0. PROPERTIES OF RULE BASES

Discussions of rule bases can be simplified by defining a set of properties describing the relationship between the sets used in the universes of discourse for the antecedents and those for the consequent. The properties discussed below generate specific classes of consequent sets from combinations of the antecedents. The different ways in which these new sets are generated are the rule properties. Different types of effects are described with illustrative examples from the algorithm In the following discussion, the term "class" is used to refer to a collection of sets.

Transformation

Rules that generate a completely new class of sets different from the inputs are called transformations. Table B-1 shows a rule base in which the consequent sets are completely different from the input sets. Transformations provide the means of converting measurement input data into likelihoods.

Table B-1
Example of Transformation

L_C Rules

C_g	M	U	QL	EL
	SE	U	U	VL
	AE	U	U	QL
		L	M	H
			C_g	

Reflection

When the action of a rule is to return one of the input classes exactly, this is called a reflection. An example of a reflection is shown in Table B-2. In this rule, along the right-directed diagonal of the rule, the consequent is a reflection of both input classes {H, M, L}. Reflection of the diagonal in a rule base is a common realization of this property.

Table B-2
Example of Reflection

G_R Rules

q'''	H	M	M	H
	M	L	M	M
	L	L	L	M
		L	M	H

Liquid Fraction F_T

Intensification

A rule that generates a consequent with greater intensity in some attribute than either or both of the antecedents is called an intensifier. An example of an intensifier is shown in Table B-3. When two antecedents with membership in H (high) sets are combined, the membership for the consequent in an intensified VH (very high) set is implied (see shaded element). Intensifiers are used to increase the strength of inferences when inputs are supportive or quality is high.

Table B-3
Example of Intensification

G Rules

G_T	H	H	H	VH
	M	M	M	H
	L	L	L	M
		L	M	H

G_R

Relaxation

Rules that imply a consequent with less intensity in some attribute than in either or both of the inputs is called a relaxation. An example of a relaxation is shown in Table B-4. When a very intense input, with membership in EL (extremely likely) and a contradictory but still very intense antecedent with membership in VU (very unlikely) are combined (shaded element), consequent membership in a diluted set U (unresolved) is implied. Relaxations are used to reduce the intensity of inferences when data are contradictory or quality is low.

Table B-4
Example of Relaxation

L_F Rules

L_C	EL	U	U	EL	EL	EL
	U	VU	QU	U	QL	VL
	EU	EU	EU	EU	U	U
		VU	QU	U	QL	VL

L_{PE}

Expansion

When a rule generates an output class that is a superset of one or both of the input universes of discourse, the property is called an expansion. An example of an expansion is the rule base shown in Table B-5. In this example, combining the vector of sets for χ expands the vector of likelihood sets $L_{\delta h\theta}$

from {U, QL, VL} to {U, QL, VL, EL}. EL does not exist in the universe of discourse for $L_{\delta h\theta}$ but does exist in the output, so the rule that combines H and VL to produce EL is an expansion property. An expander is actually a special case of an intensifier that includes sets outside the range of the input class.

Table B-5
Example of Expansion

L_D Rules

c	H	U	VL	EL
	M	U	QL	VL
	L	U	U	QL
		U	QL	VL
			L_{dhq}	

Compression

When an input class is converted into one of its subsets by a rule, the rule is called a compression. An example of a compression is the rule shown in Table B-6. In this example, the effect of membership in the set VL for one antecedent on the other antecedent is to compress the vector of likelihood sets L_{PE} from {VU, QU, U, QL, VL} to { VL, U}. This is seen in either the last column or first row in the rule table. A compressor is a special case of a reducer that reduces the range of the input class when generating an output class.

Table B-6
Example of Compression

L_{PE} Rules

L_P	VL	U	U	VL	VL	VL
	QL	U	U	QL	VL	VL
	U	VU	QU	U	QL	VL
	QU	VU	VU	QU	U	U
	VU	VU	VU	VU	U	U
		VU	QU	U	QL	VL
				L_E		

2. RULE TAXONOMY

The rule properties above define what characteristics the class of sets used in the universe of discourse for the consequent possesses given the classes used to express the antecedents. Another way to consider the nature of rule bases is to classify them according to the types of antecedents. There are two basic types, conflation and convolution rule bases.

2.1. Conflation Rule Bases

Conflation rule bases are used to combine dissimilar inputs to produce an inference. This type of rule base is used when inferences are best made by combining the input from several related variables, rather than by drawing separate inferences from each variable separately. Rule bases in the FGWL screening AR model that are of the general conflation type are shown in Table B-7. Various specific subtypes of conflation rules are used in the AR model, and these will be discussed individually below. This type of rule is used when several different inputs provide similar inferential potential, but their aggregate is more definitive than any one alone. Conflation rules typically show transformation properties.

Table B-7
Conflation Rules

Function	Input 1	Input 2	Output
Evaluate Quality of ΔH Estimates	M_{81}	M_E	Q
Combine Dome and Waste Dynamics Likelihoods with Maximum Concentration	L_{DW}	C_M	L_1
Combine Slope Probability and Correlation Coefficient	P_{Si}	R_i^2	P_{RPi}
Combine Gas Probability and Slope	P_{RPi}	S_i	L_{PRSi}

Phenomenological Rules

In a phenomenological rule base, the antecedent variables are related according to a model of the phenomena. Based on the inputs, an inference is generated using the model. This type of rule base is needed when generating inferences about gas retention and generation in the enabler segment of the model. This type of rule contains the model implicitly in its logic. The linguistic expression for the consequent and its universe of discourse are a direct reflection of the phenomenology contained in the model. The rule bases that are of this type are shown in Table B-8. Phenomenological rules almost always exhibit transformation properties.

Table B-8
Phenomenological Rules

Model	Input 1	Input 2	Output
Liquid Gas Retention	C_O	S	R_L
Salt Cake Gas Retention	Φ	F_I	R_S
Thermal Gas Generation	C_O	T	G_T
Radiolytic Gas Generation	q''	F_T	G_R

Qualification Rules

Qualification rules combine a quality metric with another antecedent to modify the inferences that would be drawn from the second antecedent alone. This implication type is used when drawing likelihood inferences from a single physical parameter. The quality metric may be a qualitative evaluation of the inherent quality of a measurement or it may be the number of data samples in an input. The rule sets that are of the model type are shown in Table B-9. Qualification rules often show intensification and relaxation properties.

Table B-9
Qualification Rules

Function	Input	Qualifier	Output
Modify ΔH Likelihood Estimates	Δh	Q	$L_{\Delta h}$
Modify Expected Slope Estimates	L_{PRSi}	I_i	L_i
Modify Dome Space Concentration	C_g	X_g	L_{CX}
Modify Dome Space Overpressure	O	X_S	L_{CX}
Modify Dome Space Concentration Likelihood	L_{CX}	N_g	L_g
Modify Dome Space Overpressure Likelihood	L_O	N_O	L_O
Modify Short-Term Temperature Change	δθ	$N_{\delta\theta}$	L_θ
Modify Short-Term Level Change	δh	$N_{\delta h}$	$L_{\delta h}$
Modify Short-Term Level-Temperature Change	$L_{\delta h\theta}$	χ	L_W

Extensive Rules

Extensive rules combine an extensive metric with an intensive variable to produce an inference that depends not only on the value of the intensive variable but also on the value of the extensive metric. This rule is used when the amount of gas retention depends not only on intensive waste characteristics, but also on the volume of waste with those characteristics. The rule sets that are of the model type are shown in Table B-10. Extensive rules can exhibit reflection, intensification, and expansion properties.

Table B-10
Extension Rules

Function	Input 1	Extender	Output
Modify Liquid Retention by Liquid Waste Volume	R_l	V_N	R_L
Modify Solid Retention by Liquid Waste Volume	R_s	V_S	R_S

2.2. Convolution Rule Bases

Convolution rules are used to combine antecedents of the same type to infer a consequent that is also of the same type. The two kinds of linguistic variables that appear in this type of rule base are potentials and likelihoods. Convolution-type rule bases are shown in Table B-11. This type of rule frequently is used when combining likelihood inferences from many different sources of data. These rules exhibit reflection, intensification, relaxation and sometimes expansion and compression properties.

Table B-11
Convolution Rules

Function	Input 1	Input 2	Output
Combine Level and Barometric Pressure Likelihoods	$L_{\Delta h}$	L_B	L_P
Combine FIC and ENF Gas Likelihoods	L_f	L_e	L_{fe}
Combine MT and NL Gas Likelihoods	L_m	L_n	L_{mn}
Combine Barometric Pressure Gas Likelihoods	L_{fe}	L_{mn}	L_B
Combine Predictor and Enabler Likelihoods	L_P	L_E	L_{PE}
Combine Dome-Space Concentration and Overpressure Likelihoods	L_g	L_O	L_D
Combine Short-Term Level and Temperature Change Likelihoods	$L_{\delta h}$	$L_{\delta\theta}$	$L_{\delta h\theta}$
Combine Dome Space and Dynamic Likelihoods	L_D	L_W	L_{DW}
Combine Indicator and Predictor-Enabler Likelihoods	L_I	L_{PE}	L_F
Combine Gas Generation and Retention Potentials	R	G	L_E
Combine Supernate and Salt Cake Retention Potentials	R_L	R_S	R
Combine Thermolytic and Radiolytic Gas Generation Potentials	G_T	G_R	G

APPENDIX C

COMPLETE EVALUATION FOR ONE MONTE CARLO TRIAL FOR TANK U-106

In this appendix, we present the results of a single Monte Carlo trial using the entire AR model for Tank U-106. This discussion is intended as a companion to that in Sec. 5 and shows in detail how the values for the elements of evidence affect the inferences made in the various logic modules.

The evaluation of the final likelihood for the retention of significant gas, L_F, proceeds as described in Sec. 3 and as shown in Fig. C-1. We will discuss at some length the details of the evaluation for the predictor likelihood, L_P, and the enabler likelihood L_E. The qualitative factors used for determining L_I are for illustration purposes only as noted above, and therefore, L_I is discussed only briefly.

1.0. PREDICTOR LIKELIHOOD, L_P

Recall that the predictor likelihood, L_P, is determined using a convolution rule with the barometric pressure likelihood, L_B, and the long-term level change likelihood, $L_{\Delta h}$, as inputs. We begin by considering L_B.

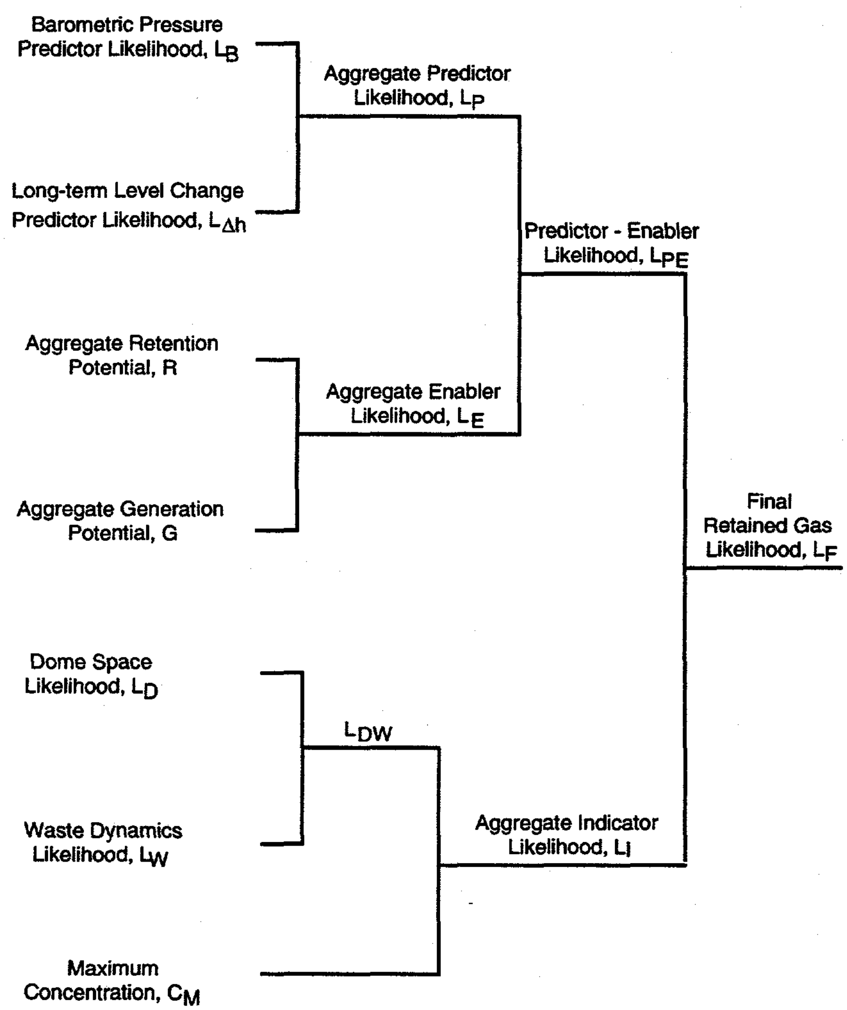


Fig. C-1. Truncated logic tree for the evaluation of L_F.

1.1. Barometric Pressure Likelihood, L_B

Figure C-2 shows the complete logic tree for this evaluation. As noted earlier, the evaluation of the branch for each instrument is identical. Figure C-3 shows the logic structure for L_f. Also shown are the input values from a single Monte Carlo trial for the primary inputs P_S, R^2, S, and I. Recall that these variables are the fraction of negative

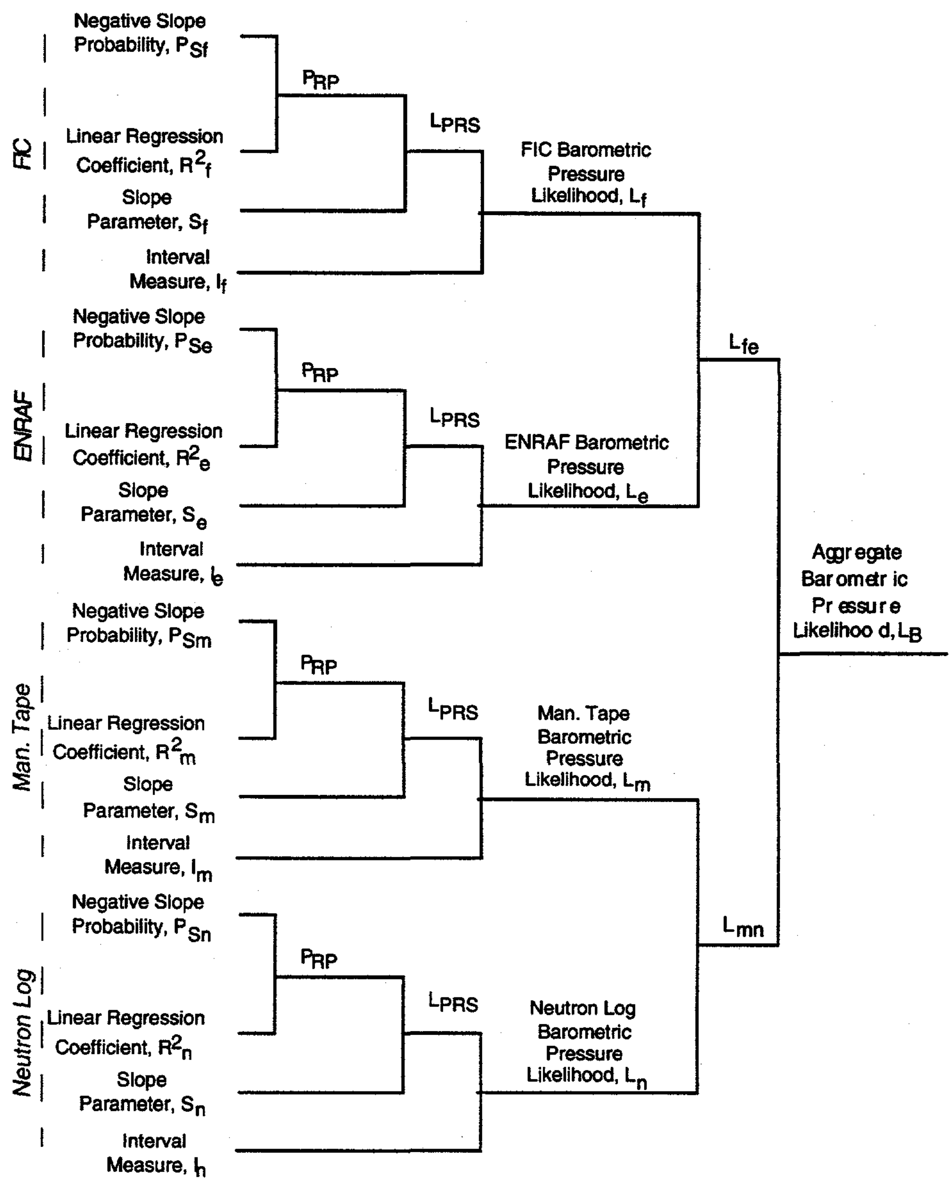


Fig. C-2. Logic structure for evaluation of barometric pressure correlation likelihood L_B.

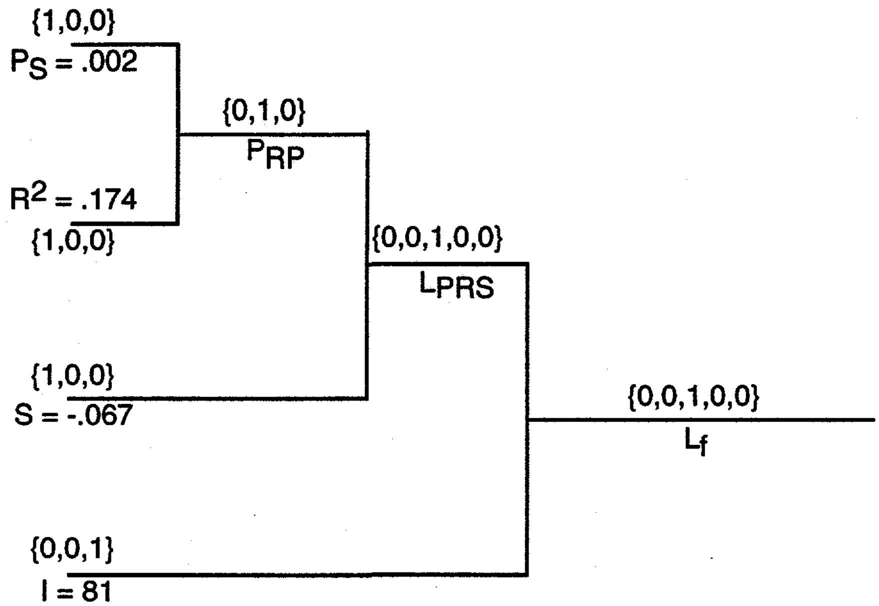


Fig. C-3. L_f with inputs and DOMs in applicable fuzzy sets.

slopes probability, the linear regression coefficient, the pressure-height slope, and the number of intervals used to calculate these statistics for the FIC sensor in Tank U-106. Also shown for each parameter in the logic tree is a set that represents the DOM in each fuzzy set in which the parameter may have membership. Set membership for primary inputs is determined directly from their defined membership functions. P_S is described by fuzzy sets {Low}, {Unresolved}, and {High} with the membership functions shown in Fig. 3-5 in the main body of the report. The value of P_S is fixed for each tank, and for Tank U-106, it is $P_S = .002$. This gives DOM in the three fuzzy sets {Low}, {Medium}, {High} of $\mu(\text{Low}) = 1$, $\mu(\text{Medium}) = 0$ and $\mu(\text{High}) = 0$. We denote this as $\gamma(P_S) = \{1,0,0\}$, which is a vector with the three elements $\mu(P_S, L) = 1$, $\mu(P_S, U) = 0$ and $\mu(P_S, H) = 0$. For this particular trial, R^2 has a value of 0.174, which translates to memberships in {Poor, Fair, Good} of $\gamma(R^2) = \{1,0,0\}$. Similarly, for S: {Positive, Slightly Negative, Very Negative}, $\gamma(S) = \{0,1,0\}$, and for I: {Small, Medium, Large} we have $\gamma(I) = \{0,0,1\}$. In this particular trial, all of the input values are such that each parameter has full membership in only one set.

The first rule evaluation in the tree is for P_{RP} using P_S and R_2 as inputs. P_{RP} is an estimate of the probability of gas and is represented by P_{RP}: {Low, Unresolved, High}. The logic structure is shown in Fig. C-3 along with the DOMs for P_S and R_2. Recall that the memberships for P_{RP} are given by the max-min operator for evaluating the rule implication (see Appendix A). The minimum membership for each rule is shown in the boxed portion of Table C-1 along with the output fuzzy set for P_{RP} generated by the implication

If P_S AND R_2 THEN P_{RS} .

For example, in the shaded box, the implication is

If P_S is *Low* AND R^2 is *Poor* THEN P_{RP} is *Unresolved*

In this case, $\mu(P_S, U) = 0$, $\mu(R^2, P) = 1$ evaluates to $\mu(P_{PR}, L) \rightarrow 0$ because $\min(\mu(P_S, U), \mu(R^2, P)) = 0$. That is, the DOM in {Low} for P_{RP} is 0. The maximum DOM for each fuzzy set for P_{RP} from the table is $\mu(L) = 0$, $\mu(U) = 1$, $\mu(H) = 0$, which we represent as $\gamma(P_{RP}) = \{0,1,0\}$. In this simple case with only full memberships in single sets for the inputs, only a single rule is active. An equivalent logic statement is

If the negative slope fraction probability is *Low* AND the linear regression coefficient is *Poor* THEN the gas probability is *Unresolved.*

This logic is consistent with a best-estimate judgment for the implication of the two inputs.

Table C-1
Numerical Example of the Evaluation of the Conflation Rule for P_S and R^2 to Generate P_{RP}

Ps	H	0	L (0)	U (0)	U (0)
	M	0	L (0)	U (0)	H (0)
	L	1	U (1)	H (0)	H (0)
			1	0	0
			P	F	G
				R^2	

Returning to Fig. C-3, the slope parameter S has a value of -0.067 for this trial and therefore from Fig. 3-5 has DOMs S: {0,1,0}. The first likelihood estimate for retained gas, L_{PRS}, is generated from the rule combining P_{RP} and S using the rule base in Table 3-4. Recall that L_{PRS} is defined on the universe $L_{PRS} \in$ {Very Unlikely, Quite Unlikely, Unresolved, Quite Likely, Very Likely}.* Full membership in {Slightly Negative} for S combines with $\gamma(P_{RP})$ = {0,1,0} to yield $\gamma(L_{PRS})$ = {0,0,1,0,0}. The logic incorporated in Table 3-4 is

> If the gas probability is *Unresolved* and the pressure-height slope is *Slightly Negative* THEN the retained gas likelihood is *Unresolved.*

Referring back to Fig. C-2, this likelihood is combined with I to obtain the FIC estimate, L_f, for the likelihood of a significant quantity of retained gas. I is defined on the universe I$\in$ {Small, Medium, Large}, and in this case, for I = 81, γ(I) = {0,0,1}. The rule for L_{PRS} and I in Table 3-5 evaluates to $\gamma(L_f)$ = {0,0,1,0,0}. That is, for the FIC, the likelihood of retained gas using the input values for this trial is represented by full membership in the unresolved fuzzy set. This result is fully consistent with the logic discussed in Sec. 3.

Figures C-4 and C-5 show the results generated from evaluation of the logic trees for the ENRAF and NL sensors. For these sensors, $\gamma(L_e)$ = {0,.65,.35,0,0} and $\gamma(L_n)$ = {0,0,.53,.29,0}. It can be seen that these results are not in good agreement. Based on the ENRAF, the likelihood has strong membership in the {Quite Unlikely} set. In fact, this judgment would be even stronger if the number of available intervals was greater as L_{PRS} for the ENRAF is $\gamma(L_e)$ = {.65,.35,0,0,0}. On the other hand, the values for P_S, R^2, and S for the neutron log generate a judgment for L_{PRS} just as strong in the opposite direction, $\gamma(L_n)$ = {0,0,0,.47,.53}. Again, the number of intervals moderates this judgment. Note that the numerical values

* Likelihoods are represented by a class of fuzzy sets with three, four, five or seven elements. The final aggregate likelihood has seven elements ranging from {Extremely Unlikely} to {Extremely Likely}. Positive indicators have four elements that range from {Unresolved} to {Extremely Likely} while a negative indicator is the reflection of this. The set of fuzzy sets for the aggregate indicator likelihood L_I has three elements, {Extremely Unlikely}, {Unresolved}, {Extremely Likely}. All other likelihoods vary from {Very Unlikely} to {Very Likely} and the set of fuzzy sets for these parameters has five elements.

for DOM always correspond to values that first are obtained from membership functions for primary inputs. For example, in $\gamma(L_n)$ = {0,0,.53,.29,0} the value 0.53 originally represented the DOM in {Very Negative} for S, and 0.29 was originally the DOM for I in {Medium}. The sets for which these values indicate DOM change as the evaluation proceeds toward the right in the logic tree. The values that "survive" are determined by the application of the min-max operation at each rule.

At this point, it is necessary to reconcile the three somewhat contradictory likelihoods. The process for doing this is shown in Fig. C-6. Note that although the manual tape instrument is not available for Tank U-106, the approach used here for missing instruments is to give them full membership in

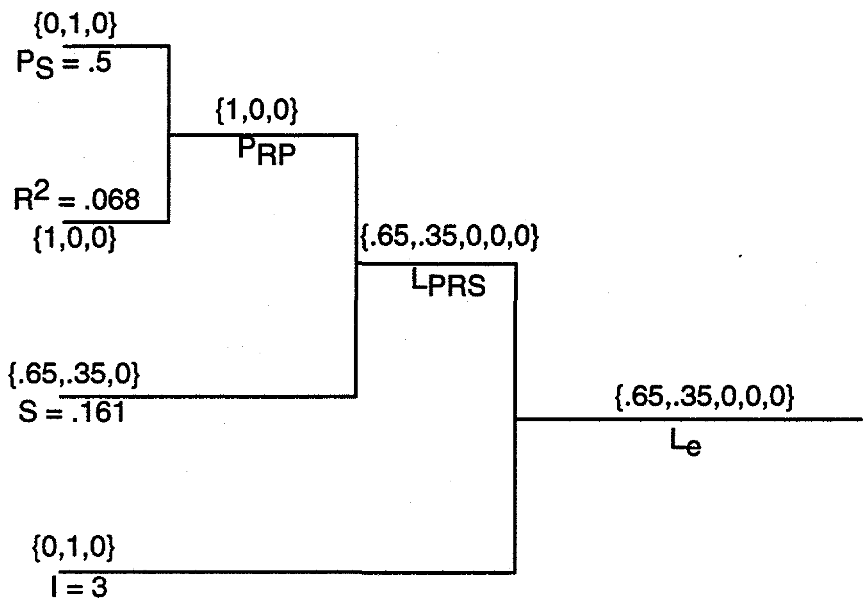


Fig. C-4. L_e with inputs and degrees of membership in applicable fuzzy sets.

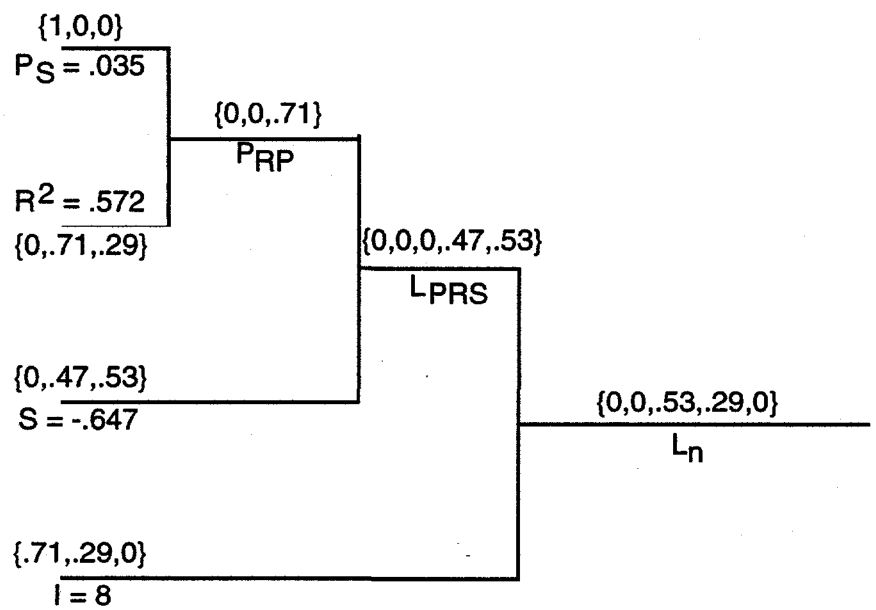


Fig. C-5. L_n with inputs and degrees of membership in applicable fuzzy sets.

{Unresolved}. For this trial, one sensor in each group mn and fe has this membership. Additionally, both rules are constructed so that if one input is $\gamma = \{0,0,1,0,0\}$, then the output likelihood for the pair replicates that of the other input. Therefore, $\gamma(L_{fe}) \rightarrow \{0,.65,.35,0,0\}$ and $\gamma(L_{mn}) \rightarrow \{0,0,.53,.29,0\}$. Finally, these two likelihoods are combined according to Table 3-8, giving a DOM of $\gamma(L_B) = \{0,.53,.35,.29,0\}$. Note that the effect of the disagreement between L_{mn} and L_{fe} reduced the DOM of L_B in {Quite Unlikely} and generated membership in {Quite Likely}.

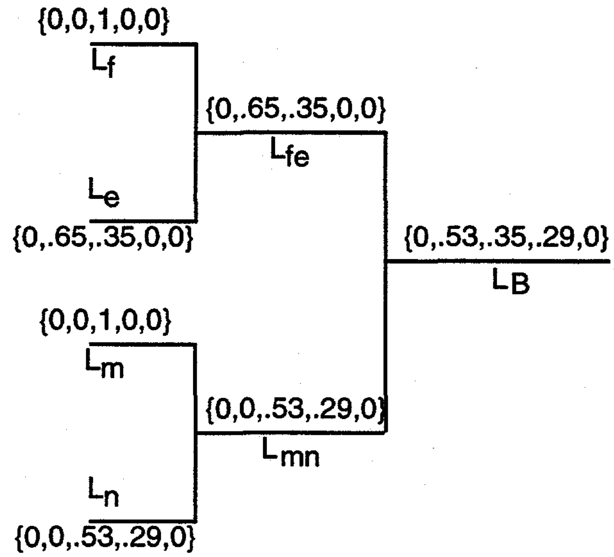


Fig. C-6. L_B with inputs and degrees of membership in applicable fuzzy sets for individual sensor and aggregate likelihoods.

1.2. Long-Term Level Change Likelihood, $L_{\Delta h}$

This estimator depends on the calculated level change, Δh, and two level ratios, M_{81} and M_E, as discussed in Sec. 3.2.2 and shown in Fig. C-7. In this trial, the ratios used to provide a numerical measure of the importance of the correction terms for pre-1981 level change and evaporation are large, which means that their role in estimating Δh is also large. According to Table 3-11, the quality, Q, associated with Δh is judged to be poor, so Q: {Poor, Fair, Good} evaluates to $\gamma(Q) = \{1,0,0\}$. In this case, the rule for $L_{\Delta h}$ is independent of the actual membership degrees for Δh. The rule truncates to

If the quality parameter is *Poor* THEN $L_{\Delta h}$ is *Unresolved*.

This is consistent with the philosophy that no definitive conclusions should be drawn from poor data.

1.3. Final Predictor Likelihood Estimate

Figure C-8 shows the combination of L_B and $L_{\Delta h}$ to produce the predictor likelihood L_P. Again because $L_{\Delta h}$ has non-zero membership only in {Unresolved}, the output of L_P is the same as for $\gamma(L_B) = \{0,.53,.35,.29,0\}$. It may appear that the level change branch of the logic tree has played no role in this evaluation. However, if the DOM for $L_{\Delta h}$ had been less than the DOM for any fuzzy set representing L_B, then the corresponding element in L_P would have been determined by $L_{\Delta h}$ rather than L_B.* This would be true regardless of the poor quality of the measurement.

* For example if $\mu(L_{\Delta h}, U) = \alpha$, $0.3 \le \alpha \le 0.35$ then L_P: $\{0,\alpha,\alpha,.29,0\}$.

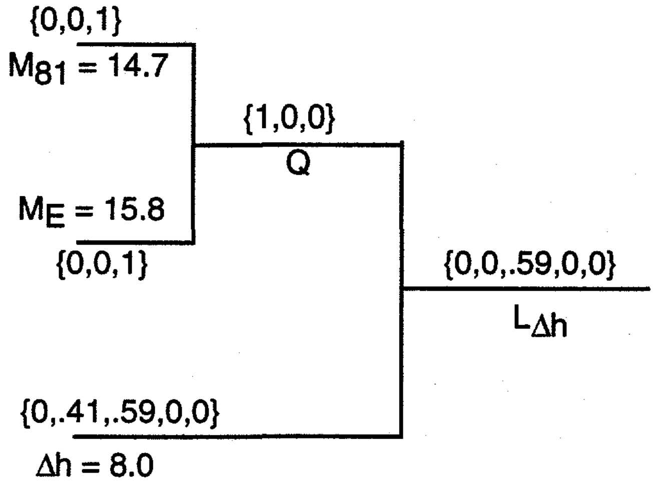


Fig. C-7. $L_{\Delta h}$ with inputs and degrees of membership in applicable fuzzy sets.

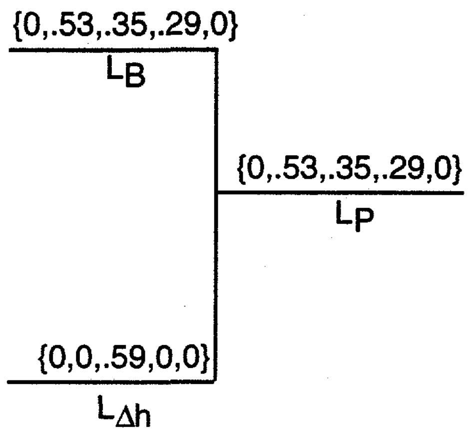


Fig. C-8. Aggregate predictor likelihood L_P with inputs L_B and $L_{\Delta h}$ and degrees of membership in applicable fuzzy sets.

2. EVALUATION OF ENABLER LIKELIHOOD, L_E

The evaluation of enabler likelihood proceeds as described in Sec. 3.3. There are two basic components of the enabler estimate—generation and retention potential. We consider the two separately.

2.1. Gas Generation Potential, G

Figure C-9 shows the logic tree for gas generation from chemical reaction and radiolysis. Chemical production depends on total organic carbon, C_O, and temperature, T. Both have universes {Low, Medium , High}. Here C_O has full membership in {High}, and $\gamma(T)$ = {.24, .76, 0}. Generation potential, G, has the universe G∈{Low, Medium, High, Very High}. For these inputs, $\gamma(G_T)$ = {0, .24, .76, 0}. Radiolysis depends on the volumetric heat generation, q''', and the liquid fraction, F_T, which both have universes {Small, Medium, Large}. For both parameters, there is membership in {Medium} and {High}. The radiolysis potential evaluates to $\gamma(G_R)$ = {0, .52, .16,0} in our example. The total gas generation potential obtained from G_R and G_T, is $\gamma(G)$ = {0, .24, .52, .16}. Note that G has μ(G,Very High) = .16. This arises from the convolution rule in Table 3-16.

If the chemical gas production potential is *High* and the radiolysis potential is *High*, THEN the total potential is *Very High*..

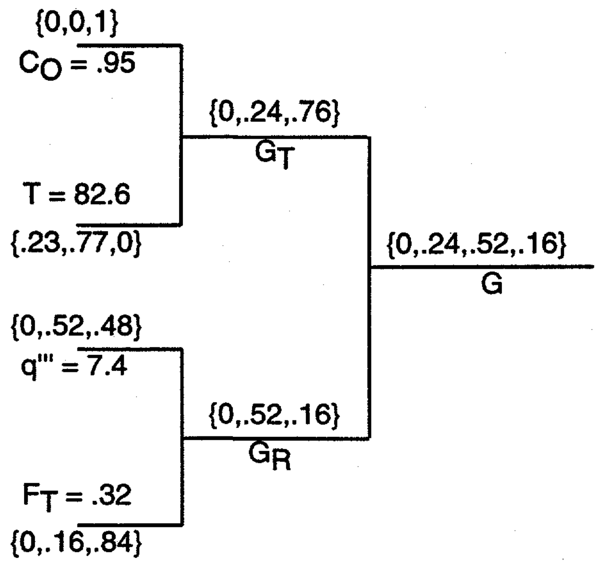


Fig. C-9. Logic tree for gas generation potential.

2.2. Gas Retention Potential, R

Figure C-10 shows the logic tree for gas retention. Recall that the potential for retention in the solids and supernate layers is evaluated. Retention in the liquid layer is assumed to depend on C_o, and the specific gravity, S. As discussed in Sec. 3.3.2.2, the amount of liquid waste is evaluated to convert the

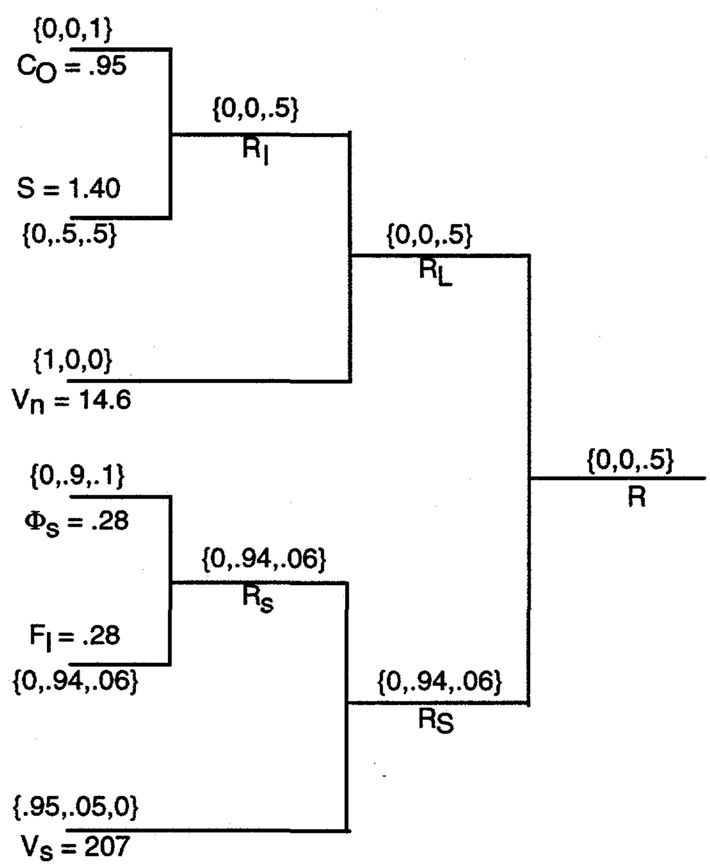


Fig. C-10. Logic tree for gas retention.

intensive retention property to an extensive capability. The extensive liquid retention, R_L has the universe {Low, Medium, High}. For the values obtained for this Monte Carlo trial, $\gamma(R_L)$ = {0, 0, .5}. Retention in the solids layers depends on the porosity and the interstitial liquid fraction. In this case, the extensive solids retention potential evaluates to $\gamma(R_S)$ = {0, .94, .06}. Finally, the aggregate retention potential is $\gamma(R)$ = {0, 0, .5}. This result occurs because the applicable rule in Table 3-18 is

> If the supernate retention potential is *High* AND the solids gas retention potential is *High* OR *Medium* THEN the aggregate retention potential is *High*.

2.3. Combination of Generation and Retention Potentials

The enabler likelihood is obtained using the conflation rule given in Table 3-23. In this case we have $\gamma(G)$ = {0, .24, .52, .16} and R: {0, 0, .5}. These inputs evaluate to $\gamma(L_E)$ = {0, 0, 0, .5, .16}. That is, the enabler likelihood has DOM in {Quite Likely} and {Very Likely}. This is certainly a reasonable conclusion given the strong potential indicated for gas generation and retention for this trial.

3. GAS INDICATOR LIKELIHOOD, L_I

For testing purposes, all of the qualitative factors used as primary inputs for the positive indicator likelihoods, L_g, L_O, $L_{\delta h}$, and L_θ, were given crisp values or distributions that would not be considered strong evidence of GRE behavior. The positive likelihoods are elements in the universe {Unresolved, Quite Likely, Very Likely, Extremely Likely}. For the inputs used here, all but L_O evaluate to γ = {1, 0, 0, 0}, whereas $\gamma(L_O)$ = {.5, 0, 0, 0}. This occurs because for this trial, the number of observations, N_O, had equal membership in {Several} and {Many}. As a result, the aggregate positive indicator likelihood evaluates to L_{DW}: {.5, 0, 0, 0}. The negative indicator C_M is an element in the universe {Very Low, Low, Medium, High}. For C_M = 83.5% of LFL, this gives DOM of $\gamma(C_M)$ = {0, 0, .33, .67}. Finally, the aggregate likelihood, L_I, can have membership in {Extremely Unlikely, Unresolved, Extremely Likely}. It is hardly surprising for the uninformative information used as test input that this evaluates to γ = {0, .5, 0}. Note that the DOM associated with L_O has been propagated to the final likelihood parameter.

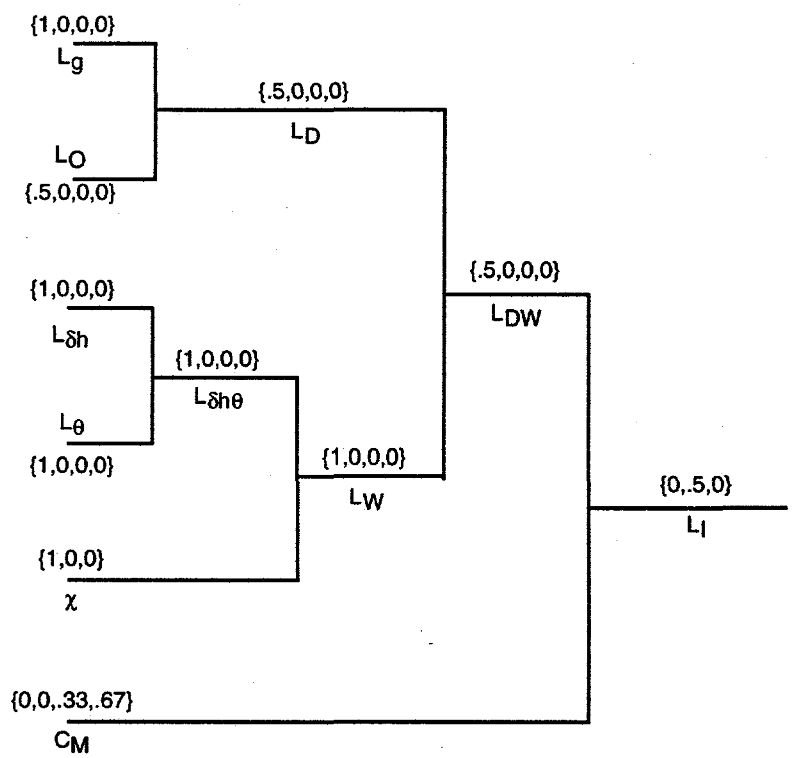


Fig. C-11. Logic tree for combining positive and negative indicator likelihoods.

4. EVALUATION OF FINAL GAS RETENTION LIKELIHOOD, L_F

The predictor, enabler, and indicator likelihoods are combined according to the rules in Sec. 3.5 with the logic structure shown in Fig. C-12. Note that L_P is L_P:{Quite Unlikely, Unresolved, Quite Likely}, whereas L_E: {Quite Likely, Very Likely}. The convolution rule for L_{PE} in this case (Table 3-35) reduces to

L_P \ L_E	QL	VL
QL	VL	VL
U	QL	VL
QU	U	U

Thus, L_{PE} can only have memberships in {Unresolved, Quite Likely, Very Likely}. The basic consideration in this rule is that both input likelihoods L_P and L_E have equal weight. If they agree, the judgment is intensified; disagreement leads to an unresolved judgment. Thus, in this case, we have L_{PE}: {0,0,.5,.35,.29}, and it can be seen that the convolution of the predictor and enabler likelihoods generates an aggregate likelihood where there is significant membership in the likelihood fuzzy sets on the likely end of the spectrum. It was seen earlier that the indicator likelihood L_I has DOM only in {Unresolved} and, according to the logic rules developed earlier, cannot affect the output of the evaluation rule. The final likelihood, L_F, is defined on the universe {Extremely Unlikely, Very Unlikely, Quite Unlikely, Unresolved, Quite Likely, Very Likely, Extremely Likely} and so must evaluate to $\gamma(L_F)$ = {0, 0, 0, .5, .35, .29, 0}. The centroid rule for defuzzification (see Sec. 3.7) is used to convert these DOMs into a crisp final result, $\lambda(L_F)$ = 0.61 in this case.

It is interesting to consider the effect of L_I if either the positive or negative indicators have membership in other than Unresolved. Suppose that we have $\gamma(C_M)$ = {.5,.5,0,0} and $\gamma(L_{DW})$ = {1,0,0,0}. That is, the calculated Quickscreen dome-space concentration is low enough to provide some membership in {Extremely Unlikely} and there are no positive indications of a GRE. In this case, $\gamma(L_I)$ = {.5,.5,0} and the final aggregate likelihood becomes $\gamma(L_F)$ = {.5,0,0,.5,.35,.29,0}. A vector with this pattern is indicative of somewhat contradictory data and the centroid value for L_F,, $\lambda(L_F)$ drops slightly to 0.59. On the other hand, if there is some positive indication of a GRE, say the DOMs are reversed with $\gamma(C_M)$ = {0,0,0,1} and $\gamma(L_{DW})$ ={0,0,.5,.5} then $\gamma(L_F)$ = {0,0,0,.5,.35,.29,.5}. In this case, the value for $\lambda(L_F)$ increases to 0.68. If the value of C_M is very small so that $\gamma(C_M)$ = {1,0,0,0} then the result is $\gamma(L_F)$ = {0,0,.5,.35,.29,0} and $\lambda(L_F)$ = 0.47. In this case, the data are totally contradictory, and no definitive judgment can be reached. Finally, if there is strong positive evidence of a GRE, the final results are $\gamma(L_F)$ = {0,0,0,0,0,0,.5}. and $\lambda(L_F)$ = 1.0. The agreement here is very strong, and the expert system provides a very clear judgment. It was noted in Sec. 3 that the memberships for L_F are asymmetric about 0.5 with a bias toward classifying tanks as "Quite Unlikely" or "Unresolved" rather than as "Very Unlikely" or "Extremely Unlikely." This reduces the power of C_M to influence L_F. If one uses a symmetric set of functions as discussed in Sec. 5, then for $\gamma(C_M)$ = {1,0,0,0} we have $\lambda(L_F)$ = 0.33 rather than 0.47. This illustrates the ability of the algorithm to incorporate different linguistic "degrees of conservatism."

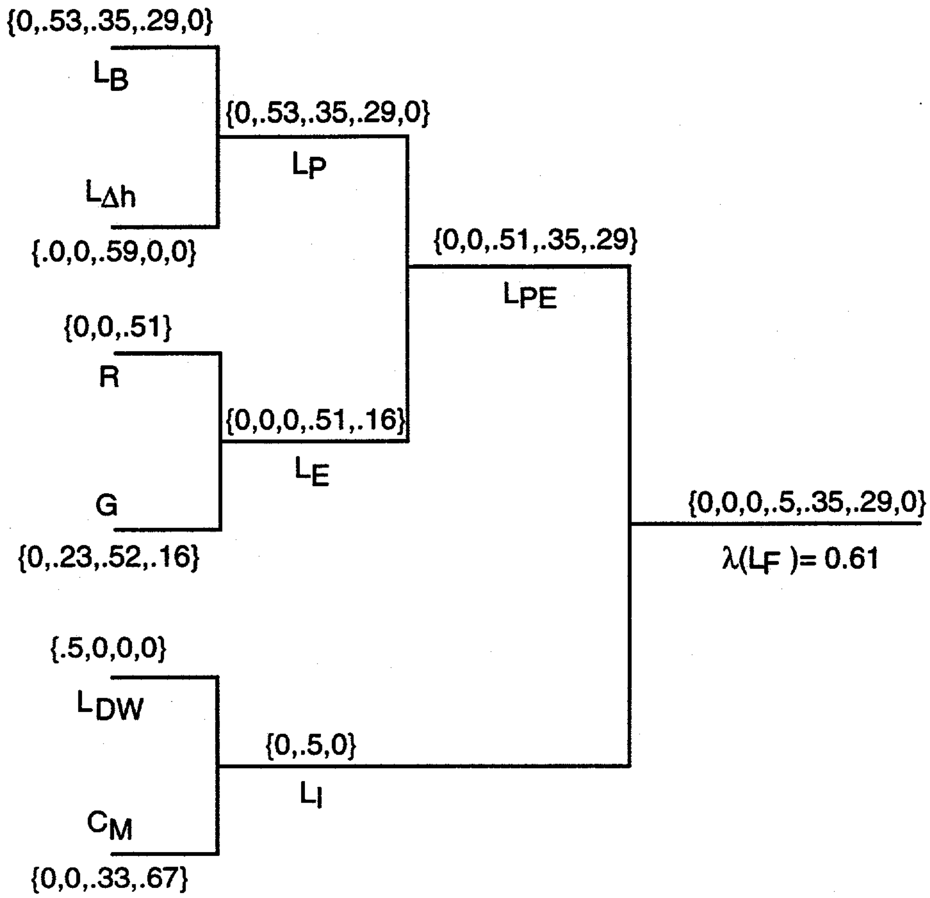


Fig. C-12. Combination of predictor, enabler, and indicator likelihoods to obtain L_F.

REFERENCES

Bacon 1996. R. Bacon, "Evaluation of Flammable Watch List Criteria," letter to J. Kinzer (March 1996).

Harmon 1991. H. Harmon, "Watchlist for Tanks Which May Have a Hydrogen Buidup," letter to R. Gerton (February 1991).

Hodgson 1995. K. M. Hodgson et al., "Evaluation of Hanford Tanks for Trapped Gas," Westinghouse Hanford Corporation report WHC-SD-WM-ER-526 (December 1995).

Hopkins 1994. J. D. Hopkins, "Criteria for Flammable Gas Watch List Tanks," Westinghouse Hanford Corporation report WHC-EP-0702 (August 1994).

Hopkins 1995. J. D. Hopkins, "Methodology for Flammable Gas Evaluations," Westinghouse Hanford Company report WHC-SO-WM-TI-274 (December 1995).

Kosko 1992. B. Kosko, *Neural Networks and Fuzzy Systems* (Prentice-Hall, Englewood Cliffs, New Jersey, 1992).

Lawrence 1990. M. Lawrence,"Unreviewed Safety Question—Waste Storage Tanks," letter to L. P. Duffy, US Department of Energy (May 1990).

Lytle 1994. J. Lytle, "Action: Delegate Authority to the Office of Waste Management to Make Modifications to the Hanford High-Level Waste Tank Watch List," letter to T. Grumbly, US Department of Energy (October 1994).

Meyers 1991. M. Meyers and J. Booker, *Eliciting and Analyzing Expert Judgment* (Academic Press, London, 1991).

Press 1992. W. H. Press, S. A. Teukolsky, W. T. Vetterling, and Brian P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, 2nd Ed., (Cambridge University Press, 1992).

Ross 1995. T. J. Ross, *Fuzzy Logic with Engineering Applications* (McGraw-Hill, New York, 1995).

Thomas 1995. S. F. Thomas, *Fuzziness and Probability* (ACG Press, Wichita, Kansas, 1995).

WHC 1990. "Operating Specifications for Watch List Tanks," Westinghouse Hanford Corporation report OSD-T-151-00030 (1990).

Whitney 1995. P. Whitney, "Screening the Hanford Tanks for Trapped Gas," Pacific Northwest Laboratory report PNL-10821 (October 1995).

Zadeh 1965. L. Zadeh, "Fuzzy Sets," Information and Control **8**, 335–353 (1965).

Zadeh 1975a. L. Zadeh, "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning—I," Information Science **8**, 199–249 (1975).

Zadeh 1975b. L. Zadeh, "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning—II," Information Science **8**, 301–357 (1975).

Zadeh 1976. L. Zadeh, "A Theory of Approximate Reasoning," in *Machine Intelligence*, J. Hayes, D. Michie, and L. Mikulich, Eds. (Halstead Press, New York, 1976), pp. 149–194.